UNIT - I

II-(1)

TRANSMISSION LINE THEORY

NEED FOR TRANSMISSION LINES:-

DEFINITION OF TRANSMISSION LINES:

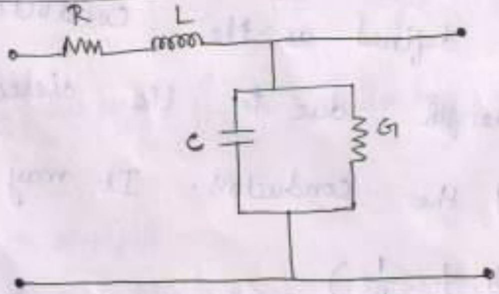
→ Transmission lines are guided conducting structures which are used in power distribution at low Frequencies, in communications and computer networks at higher Frequencies.

→ A transmission line is a means of transfer of information from one point to another.

TYPES OF TRANSMISSION LINES

1) Two wire Parallel lines
2) coaxial lines
3) Twisted wires
4) Parallel plates or planar lines
5) wire above conduction line
6) Microstrip lines
7) optical fibres

EQUIVALENT CIRCUIT OF A PAIR OF TRANSMISSION LINES



→ The Equivalent circuit consists of Cascaded Sections and each Section consists of a Series resistance R, series Inductance L, Shunt Capacitance C and Shunt conductance G.

→ R is expressed in ohm/unit Length

→ L in Henry/unit length, C in Farad/unit Length and G in Mho Per unit Length.

PRIMARY CONSTANTS:

→ The R (Ω/km), L (H/km), C (F/km) and G (mho/km) are known as Primary constants.

SALIENT ASPECTS OF PRIMARY CONSTANTS

1) R is defined as Loop resistance per unit length of line.

2) L is defined as loop Inductance per unit Length

3) C is defined as shunt Capacitance between two wires per unit length.

4) G is defined as the Conductance per unit length due to the dielectric medium separating the conductors. It may noted that ($G \neq \frac{1}{R}$)

5) R, L, C, G are distributed along the

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6) For each line the conductors are characterised by σ_c and $\mu_c = \mu_0$, $\varepsilon_c = \varepsilon_0$, and the dielectric medium, which is basically homogeneous, separating the conductor is characterised by σ_d, μ_d, ε_d

7) R, L, C & G depend on the geometry of transmission line; characteristics of the dielectric material and in some cases on the frequency.

SECONDARY CONSTANTS

The secondary constants are

1) Propagation constant (γ)
2) Characteristic impedance (Z_0)

Propagation Constant

Def 1 $\gamma = \sqrt{\text{Series Impedance} \times \text{shunt admittance}}$

$$= \sqrt{ZY} \quad (1/m)$$

Def 2 $\gamma = \log_e \left(\frac{I_S}{I_L} \right) = \log_e \left(\frac{V_S}{V_L} \right)$ nepers

Def 3 $\gamma = 20 \log_{10} \left(\frac{I_S}{I_L} \right) = 20 \log_{10} \left(\frac{V_S}{V_L} \right)$ dB

Def 4 $\gamma = \alpha + j\beta$

$\alpha \rightarrow$ Attenuation constant, db/m

$\beta \rightarrow$ Phase constant, rad/m.

II - 4.

1 neper = 8.686 db.

1 rad = 57.3°.

We know $\alpha = R + j\omega L$

$\beta = G + j\omega C$.

$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\gamma^2 = (\alpha + j\beta)^2 = (R+j\omega L)(G+j\omega C)$

$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta = RG + jR\omega C + jG\omega L - \omega^2 LC$

$= (RG - \omega^2 LC) + j(\omega RC + \omega LG)$

Taking real part

$\alpha^2 - \beta^2 = RG - \omega^2 LC \rightarrow (1)$

$|\gamma| = |\alpha + j\beta|$

$= \sqrt{\alpha^2 + \beta^2} = \left|\sqrt{(R+j\omega L)(G+j\omega C)}\right|$

Square root cancel on both sides $\alpha^2 + \beta^2 = |(R+j\omega L)(G+j\omega C)|$

$\alpha^2 + \beta^2 = \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)} \rightarrow (2)$

Add (1) & (2)

$\alpha^2 - \cancel{\beta^2} + \alpha^2 + \cancel{\beta^2} = (RG - \omega^2 LC) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}$

$2\alpha^2 = (RG - \omega^2 LC) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}$

$\alpha = \sqrt{\frac{1}{2}(RG - \omega^2 LC) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}}$

Subtract (1) & (2)

$\cancel{\alpha^2} - \beta^2 - \beta^2 - \cancel{\alpha^2} = (RG - \omega^2 LC) - \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}$

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$$\beta^2 = -\frac{1}{2}\left[(RG - \omega^2 LC) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \right]$$

$$\beta^2 = \frac{1}{2}\left[(\omega^2 LC - RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \right]$$

$$\boxed{\beta = \sqrt{\frac{1}{2}\left[\omega^2 LC - RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

CHARACTERISTIC IMPEDANCE (Z_0)

Def 1: The characteristic impedance Z_0 of a line is defined as the ratio of the forward voltage wave to the forward current wave at any point on the line,

$$Z_0 = \frac{V_f}{I_f} \ \Omega$$

Def 2 Z_0 is also defined as the ratio of the square root of series impedance to the square root of shunt admittance

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Def 3 Z_0 is defined as the minus of the ratio of the reflected voltage wave to the reflected current wave at any point on the line

$$Z_0 = -\frac{V_r}{I_r}$$

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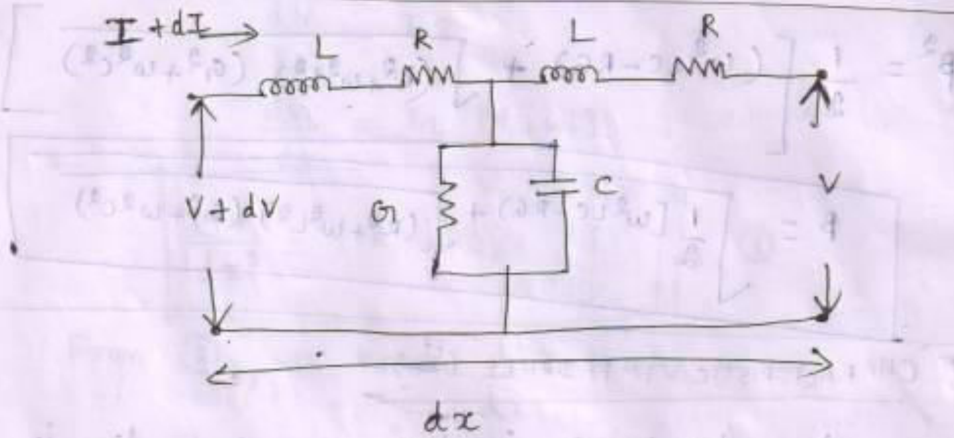
TRANSMISSION LINE EQUATION - GENERAL SOLUTION



→ Transmission line is a conductive method of guiding electrical energy from one place to another.

→ A uniform transmission line can be considered to be made up of an infinite number of T sections each of infinitesimal size, dx

→ The Equivalent circuit of T section of transmission line is

→ The Parameters R, L, G and C are distributed throughout the transmission line.

→ The series impedance per unit length and Shunt admittance per unit length are:

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

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→ Consider a T section of transmission line of length dx.

→ Let $V + dV$ be the voltage and $I + dI$ be the current at one end of T Section

→ Let V be the voltage. I be the current

→ Series impedance is given by $(R+j\omega L)\,dx$.

→ shunt admittance is given by $(G+j\omega C)\,dx$.

voltage drop across the series impedance of T section is ~~given by the potential difference between the two ends of~~ T Section

(i.e) $V + dV - V = I(R + j\omega L)\,dx$

$$\frac{dV}{dx} = I\,(R+j\omega L)$$

$$\frac{dV}{dx} = IZ \rightarrow (1)$$

current difference b/w the two ends of T Section is due to the voltage drop across the shunt admittance

$$I + dI - I = V(G + j\omega C)\,dx$$

$$dI = V(G + j\omega C)\,dx$$

$$\frac{dI}{dx} = V\,(G + j\omega C)$$

$$\boxed{\frac{dI}{dx} = VY \rightarrow (2)}$$

Taking equation 1

$$\frac{dV}{dx} = IZ$$

$$\frac{dV}{dx} = I(R+j\omega L)$$

$$\boxed{\frac{d^2V}{dx^2} = (R+j\omega L) \cdot \frac{dI}{dx}} \rightarrow 3$$

From 2 we know $\frac{dI}{dx} = VY$

$$\frac{dI}{dx} = V(G+j\omega C)$$

using the value of $\frac{dI}{dx}$ in equation 3

$$\boxed{\frac{d^2V}{dx^2} = (R+j\omega L)\ V(G+j\omega C)} \rightarrow 4$$

Taking equation 2,

$$\frac{dI}{dx} = VY = V(G+j\omega C)$$

$$\frac{d^2I}{dx^2} = (G+j\omega C)\frac{dV}{dx}$$

From equation 1 we know $\frac{dV}{dx} = IZ$

$$\frac{dV}{dx} = I(R+j\omega L)$$

$$\boxed{\frac{d^2I}{dx^2} = (G+j\omega C)\ (R+j\omega L)\ I} \rightarrow 5$$

II- 9

Propagation constant is given by

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{ZY} \rightarrow (6)$$

use the value of γ in equation 6 in 4 & 5

$$\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)\cdot V = \gamma^2 V$$

Similarly $\frac{d^2I}{dx^2} = \gamma^2 I$

$$\boxed{\begin{aligned} \frac{d^2V}{dx^2} &= \gamma^2 V \\ \frac{d^2I}{dx^2} &= \gamma^2 I \end{aligned}} \begin{aligned} &\rightarrow (7)(a) \\ &\rightarrow (7)(b) \end{aligned}$$

The Solutions of the above linear differential equations are

$$V = Ae^{\gamma x} + Be^{-\gamma x} \rightarrow (8)$$

$$I = Ce^{\gamma x} + De^{-\gamma x} \rightarrow (9)$$

where A, B, C & D are arbitrary constants.

Differentiating the equation (8) w.r. to x

$$\frac{dV}{dx} = A\gamma e^{\gamma x} - \gamma Be^{-\gamma x}$$

But $\frac{dV}{dx} = IZ$

$$IZ = A\gamma e^{\gamma x} - \gamma Be^{-\gamma x} \quad (\because \gamma = \sqrt{ZY})$$

$$IZ = A\sqrt{ZY}\, e^{\sqrt{ZY}x} - B\sqrt{ZY}\, e^{-\sqrt{ZY}x}$$

$$\boxed{I = \frac{A}{Z}\sqrt{ZY}\, e^{\sqrt{ZY}x} - \frac{B}{Z}\sqrt{ZY}\, e^{-\sqrt{ZY}x}} \rightarrow (10)$$

Differentiating equation 9 w.r. to 'x'

$$\frac{dI}{dx} = C\gamma e^{\gamma x} - D\gamma e^{-\gamma x}$$

But $\frac{dI}{dx} = VY$

$$VY = C\gamma e^{\gamma x} - D\gamma e^{-\gamma x}.$$

$$= C\sqrt{ZY}\, e^{\sqrt{ZY}x} - D\sqrt{ZY}\, e^{-\sqrt{ZY}x}$$

$$\boxed{V = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}\,x} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}\,x}} \rightarrow 11$$

The distance $x=0$ as it is measured from the receiving end of the transmission line

$$\therefore I = I_R,\ V = V_R,\ V_R = I_R Z_R$$

$I_R \rightarrow$ Current in the receiving end of line

$V_R \rightarrow$ Voltage across the receiving end

$Z_R \rightarrow$ Impedance of receiving end

Substituting the above Conditions in eq 8, 9, 10, 11

then

8 becomes $V_R = A + B \quad (\because x = 0)$ $\rightarrow$ 12

9 becomes $I_R = C + D \quad (\because x = 0)$ $\rightarrow$ 13

10 becomes $I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \rightarrow$ 14

II - 11

11 becomes $V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \rightarrow 15$

To Solve these equations

$$\text{let } x = \sqrt{\frac{Z}{Y}} \text{ and } \frac{1}{x} = \sqrt{\frac{Y}{Z}}$$

use the above condition in equation 14.

$$I_R = \frac{A}{x} - \frac{B}{x} = \frac{1}{x}(A-B)$$

But from 13 $I_R = C+D$

$$C+D = \frac{1}{x}(A-B)$$

$$Cx + Dx = A-B$$

$$\boxed{A-B = Cx + Dx} \rightarrow 16$$

Equation 15 becomes $V_R = Cx - Dx$

But $V_R = A+B$.

$$\boxed{A+B = Cx - Dx} \rightarrow 17$$

Adding equation 16 & equation 17

$$A - \cancel{B} + A + \cancel{B} = Cx + \cancel{Dx} + Cx - \cancel{Dx}$$

$$\cancel{2}A = \cancel{2}Cx$$

$$\boxed{A = Cx} \rightarrow 18$$

Subtracting equation 16 and 17

$$\cancel{A} - B - \cancel{A} - B = \cancel{Cx} + Dx - \cancel{Cx} + Dx$$

$$-\cancel{2}B = \cancel{2}Dx \; ; \; \boxed{B = -Dx} \rightarrow 19$$

using the values of A and B in equation 12

II - (12)

$$V_R = A + B$$

$$V_R = C_x - D_x \rightarrow (20)$$

But $I_R = C + D$

$$I_R x = C_x + D_x \rightarrow (21)$$

Adding the equation (20) & (21)

$$I_R x + V_R = C_x - \cancel{D_x} + C_x + \cancel{D_x}$$

$$I_R x + V_R = 2C x$$

$$Cx = \frac{I_R x + V_R}{2}$$

$$C = \frac{I_R x + V_R}{2x}$$

$$C = \frac{I_R}{2} + \frac{V_R}{2x}$$

We know $x = \sqrt{\frac{Z}{Y}}$

$$\therefore \boxed{C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \rightarrow (22)$$

Subtracting the equation (20) & (21)

$$I_R x - V_R = \cancel{C_x} + D x - \cancel{C_x} + D x$$

$$2Dx = I_R x - V_R$$

$$Dx = \frac{I_R x}{2} - \frac{V_R}{2}$$

$$D = \frac{I_R}{2} - \frac{V_R}{2x}$$

$$\boxed{D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \rightarrow (23)$$

II- 13

But we know $A = C x$

$$\therefore C = \frac{A}{x}$$

using the above result in equation 22

$$(ie) \quad C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$\frac{A}{x} = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$\therefore A = \frac{I_R}{2}x + \left(\frac{V_R}{2}\sqrt{\frac{Y}{Z}}\; x\right)$$

$$A = \frac{I_R}{2}\sqrt{\frac{Z}{Y}} + \frac{V_R}{2}\left(\sqrt{\frac{Y}{Z}}\sqrt{\frac{Z}{Y}}\right)$$

$$\boxed{A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \rightarrow 24$$

But $B = -Dx$ using the above result in eq 23

$$D = -\frac{B}{x}$$

$$-\frac{B}{x} = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$-B = \frac{I_R x}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}\; x$$

$$B = -\frac{I_R}{2}x + \frac{V_R}{2}\sqrt{\frac{Y}{Z}\times\frac{Z}{Y}}$$

$$\boxed{B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \rightarrow 25$$

The characteristic impedance is defined as

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

II - 14

$$A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$A = \frac{V_R}{2} + \frac{I_R}{2} Z_0 \quad \left(\because V_R = I_R Z_R \right)$$

$$A = \frac{V_R}{2} + \frac{V_R}{2 Z_R} Z_0 \qquad I_R = \frac{V_R}{Z_R}$$

$$\therefore \boxed{A = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]} \rightarrow (26)$$

$$B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{V_R}{2} - \frac{V_R}{2 Z_R} Z_0$$

$$\boxed{B = \frac{V_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]} \rightarrow (27)$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$C = \frac{I_R}{2} + \frac{I_R Z_R}{2 Z_0}$$

$$\therefore \boxed{C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]} \rightarrow (28)$$

$$D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$D = \frac{I_R}{2} - \frac{I_R Z_R}{2 Z_0}$$

$$\therefore \boxed{D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]} \rightarrow (29)$$

Substituting the values of A, B, C, D in eq (8) (9)

II - (15)

$$V = \frac{V_R}{2}\left(1+\frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\left(1-\frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,x}$$

$$\boxed{V = \frac{V_R}{2}\left[\left(1+\frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}\,x} + \left(1-\frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,x}\right]} \quad \rightarrow (30)$$

(9) becomes

$$I = \frac{I_R}{2}\left(1+\frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\left(1-\frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,x}$$

$$\boxed{I = \frac{I_R}{2}\left[\left(1+\frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}\,x} + \left(1-\frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,x}\right]} \quad \rightarrow (31)$$

After simplification using equation (30) & (31)

$$V = \frac{V_R}{2}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\frac{Z_0}{Z_R}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}e^{-\sqrt{ZY}\,x} - \frac{V_R}{2}\frac{Z_0}{Z_R}e^{-\sqrt{ZY}\,x}.$$

$$I = \frac{I_R}{2}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\cdot\frac{Z_R}{Z_0}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}e^{-\sqrt{ZY}\,x} - \frac{I_R}{2}\frac{Z_R}{Z_0}e^{-\sqrt{ZY}\,x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + \frac{I_R \cancel{Z_R} Z_0}{2\cancel{Z_R}}e^{\sqrt{ZY}\,x} - \frac{I_R \cancel{Z_R} Z_0}{2\cancel{Z_R}}e^{-\sqrt{ZY}\,x}$$

$$\boxed{V = V_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + I_R Z_0\left(\frac{e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}}{2}\right)}$$

II - (16).

$$I = I_R \left(\frac{e^{\sqrt{zy}x} + e^{-\sqrt{zy}x}}{2} \right) + \frac{V_R}{z_0} \left(\frac{e^{\sqrt{zy}x} - e^{-\sqrt{zy}x}}{2} \right) \quad (33)$$

Then Equations can be written in terms of hyperbolic functions.

$$V = V_R \cosh \sqrt{zy}\, x + I_R z_0 \sinh \sqrt{zy}\, x \quad (34)$$

$$I = I_R \cosh \sqrt{zy}\, x + \frac{V_R}{z_0} \sinh \sqrt{zy}\, x \quad (35)$$

These are the equations for voltage and current of a transmission line at any distance 'x' from the receiving end of transmission line.

→ The Equations for voltage and current at the Sending end of a transmission line of length 'l' are given by

$$V_S = V_R \cosh \sqrt{zy}\, l + \frac{V_R}{I_R} z_0 \sinh \sqrt{zy}\, l \quad (36) \quad \because \left(I_R = \frac{V_R}{Z_R} \right)$$

$$I_S = I_R \cosh \sqrt{zy}\, l + \frac{I_R Z_R}{z_0} \sinh \sqrt{zy}\, l \quad (37) \quad (\because V_R = I_R Z_R)$$

The Final general Solutions are:

$$V_S = V_R \left[\cosh \sqrt{zy}\, l + \frac{Z_0}{Z_R} \sinh \sqrt{zy}\, l \right]$$

$$I_S = I_R \left[\cosh \sqrt{zy}\, l + \frac{Z_R}{} \sinh \sqrt{zy}\, l \right]$$

II - (17).

PHYSICAL SIGNIFICANCE OF THE EQUATION FOR INFINITE LINE

INPUT IMPEDANCE

The Equations for voltage and current at the sending end of a transmission line of length 'l' is given by.

$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)$$

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)$$

The input impedance of the transmission line is

$$Z_S = \frac{V_S}{I_S} = \frac{V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{\cancel{I_R}\,Z_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{\cancel{I_R}\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{\cancel{Z_R}\left(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l\right)Z_0}{\cancel{Z_R}\left(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{Z_0\left(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l\right)}{\left(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l\right)}$$

let $\sqrt{ZY} = \gamma$

$$Z_S = \frac{Z_0\left(Z_R\cosh\gamma l + Z_0\sinh\gamma l\right)}{\left(Z_0\cosh\gamma l + Z_R\sinh\gamma l\right)}$$

$$Z_S = \frac{Z_0\,\cancel{\cosh\gamma l}\left[Z_R + Z_0\frac{\sinh\gamma l}{\cosh\gamma l}\right]}{}$$

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$$Z_S = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

$\left(V_R = I_R Z_R,\ \frac{V_R}{Z_R} = I_R \right)$

From equation (30) & (31)

$$V_S = \frac{V_R}{2} \left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY} l} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY} l} \right]$$

$$= \frac{V_R}{2} \left[\left(\frac{Z_R + Z_0}{Z_R}\right) e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R}\right) e^{-\sqrt{ZY} l} \right]$$

$$V_S = \left(\frac{V_R}{2}\right) \left(\frac{Z_R + Z_0}{Z_R}\right) \left[e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l} \right]$$

$$I_S = \frac{I_R}{2} \left[\left(1 + \frac{Z_R}{Z_0}\right) e^{\sqrt{ZY} l} + \left(1 - \frac{Z_R}{Z_0}\right) e^{-\sqrt{ZY} l} \right]$$

$$I_S = \frac{I_R}{2} \left[\frac{(Z_R + Z_0)}{Z_0} e^{\sqrt{ZY} l} + \frac{(Z_0 - Z_R)}{Z_0} e^{-\sqrt{ZY} l} \right]$$

The input impedance of the transmission line is given by

$$Z_S = \frac{V_S}{I_S} = \frac{\left(\frac{V_R}{\cancel{2}}\right) \left(\frac{Z_R + Z_0}{Z_R}\right) \left[e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l} \right]}{\left(\frac{I_R}{\cancel{2}}\right) \left(\frac{Z_R + Z_0}{Z_0}\right) \left[e^{\sqrt{ZY} l} + \left(\frac{Z_0 - Z_R}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l} \right]}$$

$$= \frac{\cancel{Z_R} \cancel{(Z_R + Z_0)} Z_0}{\cancel{Z_R} \cancel{(Z_R + Z_0)}} \left[\frac{e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l}}{e^{\sqrt{ZY} l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l}} \right]$$

$$Z_S = Z_0 \left[\frac{e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l}}{e^{\sqrt{ZY} l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l}} \right]$$

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$$\therefore Z_S = Z_0 \left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}} \right]$$

→ If the line is terminated with its characteristic impedance (ie) $Z_R = Z_0$, then the input impedance becomes equal to its characteristic impedance $Z_S = Z_0$

→ The input impedance of an infinite line is determined by letting $l \to \alpha$

$$\therefore Z_S = Z_0.$$

→ It is found that a line of finite length, terminated with its characteristic impedance appears to the transmitting end generator as an infinite line.

→ A finite line terminated with Z_0 and an infinite line are same by measurements at the source.

If $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$ then

$$\boxed{Z_S = Z_0 \left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right]}$$

Lossless Transmission lines

A transmission line is

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TRANSMISSION LINE PARAMETERS

INTRODUCTION:

The electrical lines which are used to transmit the electrical waves along them are called transmission lines. Transmission line parameters are resistance, Inductance and capacitance and conductance (R, L, C and G). The transmission parameters are distributed all along the length of the transmission line. The end to which the voltage is applied is called Sending end while the end at which the signals are received is called receiving end of the transmission line.

A LINE OF CASCADED T-SECTIONS: T Section equivalent for a line of length 'l'.

T-SECTION VALUES INTERMS OF Z_0 and γ

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}\ ; \quad \frac{Z_1}{2Z_2} = (\cosh\gamma - 1)\ ; \quad \frac{Z_1}{Z_2} = 2(\cosh\gamma - 1)$$

$$\frac{Z_1}{Z_2} = 2\left(2\sinh^2\frac{\gamma}{2}\right)\ ; \quad \frac{Z_1}{Z_2} = 4\sinh^2\frac{\gamma}{2}\ ; \quad \boxed{\sinh^2\frac{\gamma}{2} = \frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}\ ; \quad \boxed{\sinh\frac{\gamma}{2} = \frac{1}{2}\sqrt{\frac{Z_1}{Z_2}}}$$

$$\sinh\frac{\gamma}{2}\cosh\frac{\gamma}{2} = \frac{Z_0}{2Z_2}\ ; \quad \cosh\frac{\gamma}{2} = \frac{Z_0}{2Z_2} \times \frac{1}{\sinh\frac{\gamma}{2}}$$

$$\cosh\frac{\gamma}{2} = \frac{Z_0}{\cancel{2}Z_2} \times \frac{1}{\frac{1}{\cancel{2}}\sqrt{\frac{Z_1}{Z_2}}}\ ; \quad \cosh\frac{\gamma}{2} = \frac{Z_0}{\cancel{\sqrt{Z_2}}\sqrt{Z_2}}\ \frac{\cancel{\sqrt{Z_2}}}{\sqrt{Z_1}}$$

$$\boxed{\cosh\frac{\gamma}{2} = \frac{Z_0}{\sqrt{Z_2 Z_1}}} \qquad \tanh\frac{\gamma}{2} = \frac{\sinh\frac{\gamma}{2}}{\cosh\frac{\gamma}{2}} = \frac{\frac{1}{2}\sqrt{\frac{Z_1}{Z_2}}}{\frac{Z_0}{\sqrt{Z_1 Z_2}}}$$

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$$\tanh\frac{\gamma}{2} = \frac{1}{2}\,\frac{\sqrt{Z_1}\,\sqrt{Z_1}\,\cancel{\sqrt{Z_2}}}{\cancel{\sqrt{Z_2}}\;Z_0} = \frac{Z_1}{2Z_0}$$

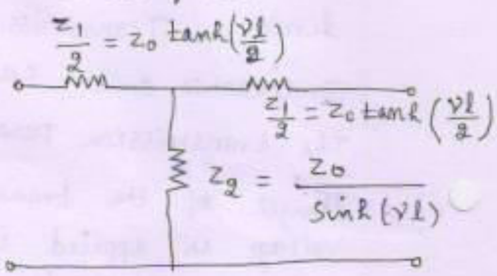
$$\tanh\frac{\gamma}{2} = \frac{Z_1}{2Z_0}$$

$$\boxed{\frac{Z_1}{2} = Z_0 \tanh\frac{\gamma}{2}}$$

$$\frac{\sinh\gamma}{\cancel{2}} = \frac{Z_0}{\cancel{2}Z_2}$$

$$\boxed{Z_2 = \frac{Z_0}{\sinh(\gamma)}}$$

T Section Equivalent



TRANSMISSION LINES:

The electrical lines which are used to transmit the electrical waves along them are called transmission lines.

TYPES OF TRANSMISSION LINES:

① open wire line ② Cables ③ co-axial line ④ Waveguides.

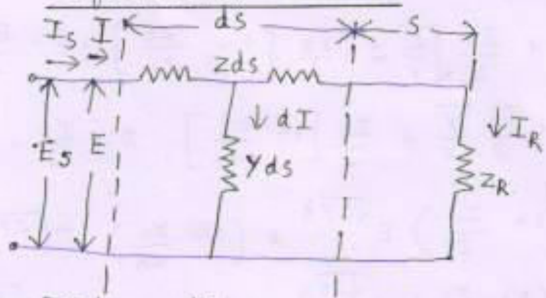
TRANSMISSION LINE PARAMETERS: The various electric parameters associated with the transmission lines are ① Resistance ② Inductance ③ capacitance ④ conductance.

GENERAL SOLUTION:

A circuit with distributed parameters requires a method of analysis somewhat different from that employed in circuits of lumped constants.

Differential circuit equations that describe this action will be written for the steady state, from which general circuit equations can be Obtained.

A Long line, with the elements of one of the infinitesimal sections.



$$dE = Izds; \quad \frac{dE}{ds} = Iz$$

$$dI = Eyds; \quad \frac{dI}{ds} = Ey$$

$$\frac{d^2E}{ds^2} = z\frac{dI}{ds}, \quad \frac{d^2I}{ds^2} = y\frac{dE}{ds}$$

$$\frac{d^2E}{ds^2} = zyE, \quad \frac{d^2I}{ds^2} = zyI$$

$$(m^2 - zy)E = 0; \quad m = \pm\sqrt{zy}$$

$$E = A\,\varepsilon^{\sqrt{zy}\,s} + B\,\varepsilon^{-\sqrt{zy}\,s}$$

$$I = C\,\varepsilon^{\sqrt{zy}\,s} + D\,\varepsilon^{-\sqrt{zy}\,s}$$

$$s = 0; \quad I = I_R, E = E_R$$

$$E_R = A + B; \quad I_R = C + D$$

$$\frac{dE}{ds} = A\sqrt{zy}\,\varepsilon^{\sqrt{zy}\,s} - B\sqrt{zy}\,\varepsilon^{-\sqrt{zy}\,s}$$

$$Iz = A\sqrt{zy}\,\varepsilon^{\sqrt{zy}\,s} - B\sqrt{zy}\,\varepsilon^{-\sqrt{zy}\,s}$$

$$I = A\sqrt{\frac{y}{z}}\,\varepsilon^{\sqrt{zy}\,s} - B\sqrt{\frac{y}{z}}\,\varepsilon^{-\sqrt{zy}\,s}$$

$$\frac{dI}{ds} = C\sqrt{zy}\,\varepsilon^{\sqrt{zy}\,s} - D\sqrt{zy}\,\varepsilon^{-\sqrt{zy}\,s}$$

$$E = C\sqrt{\frac{z}{y}}\,\varepsilon^{\sqrt{zy}\,s} - D\sqrt{\frac{z}{y}}\,\varepsilon^{-\sqrt{zy}\,s}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}}$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}}$$

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]; \quad B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]; \quad D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]$$

$$E = \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)\varepsilon^{\sqrt{ZY}s} + \left(1 - \frac{Z_0}{Z_R}\right)\varepsilon^{-\sqrt{ZY}s}\right]$$

$$I = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)\varepsilon^{\sqrt{ZY}s} + \left(1 - \frac{Z_R}{Z_0}\right)\varepsilon^{-\sqrt{ZY}s}\right]$$

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[\varepsilon^{\sqrt{ZY}s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\sqrt{ZY}s}\right]$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[\varepsilon^{\sqrt{ZY}s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\sqrt{ZY}s}\right]$$

$$E = E_R\left(\frac{\varepsilon^{\sqrt{ZY}s} + \varepsilon^{-\sqrt{ZY}s}}{2}\right) + I_R Z_0\left(\frac{\varepsilon^{\sqrt{ZY}s} - \varepsilon^{-\sqrt{ZY}s}}{2}\right)$$

$$I = I_R\left(\frac{\varepsilon^{\sqrt{ZY}s} + \varepsilon^{-\sqrt{ZY}s}}{2}\right) + \frac{E_R}{Z_0}\left(\frac{\varepsilon^{\sqrt{ZY}s} - \varepsilon^{-\sqrt{ZY}s}}{2}\right)$$

$$E = E_R \cosh\sqrt{ZY}\,s + I_R Z_0 \sinh\sqrt{ZY}\,s$$

$$I = I_R \cosh\sqrt{ZY}\,s + \frac{E_R}{Z_0}\sinh\sqrt{ZY}\,s$$

PHYSICAL SIGNIFICANCE OF THE EQUATIONS, THE INFINITE LINE:

Current I_S of a line of length l as

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)$$

If the line is terminated in $Z_R = Z_0$, then

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \sinh\sqrt{ZY}\,l\right); \quad \frac{I_S}{I_R} = \varepsilon^{\sqrt{ZY}l} = \varepsilon^{\gamma l}$$

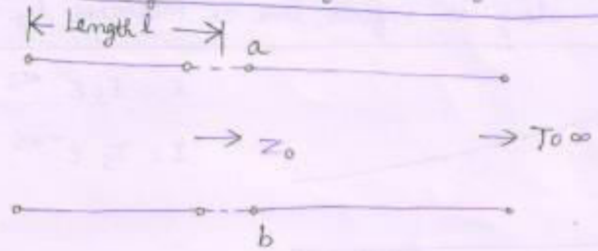
The propagation constant γ is defined per unit length of line.

$$Z_S = \frac{E_S}{I_S} = Z_0\left(\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}\right)$$

$$Z_S = \frac{E_S}{I_S} = Z_0\left[\frac{\varepsilon^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma l}}{\varepsilon^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma l}}\right]$$

Unity has been established between the lumped-constant and distributed constant circuits in that the description of circuit performance through Z_0 and γ has been found to hold for both cases and thus is a truly general method of description of circuit performance.

A length l taken from an infinite line



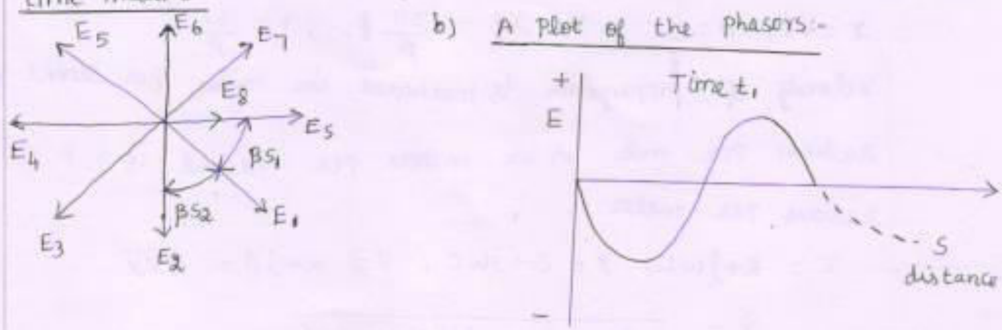
$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[\varepsilon^{\gamma s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma s}\right]$$

$$I = \frac{I_R(Z_R + Z_0)}{2Z_0}\left[\varepsilon^{\gamma s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma s}\right]$$

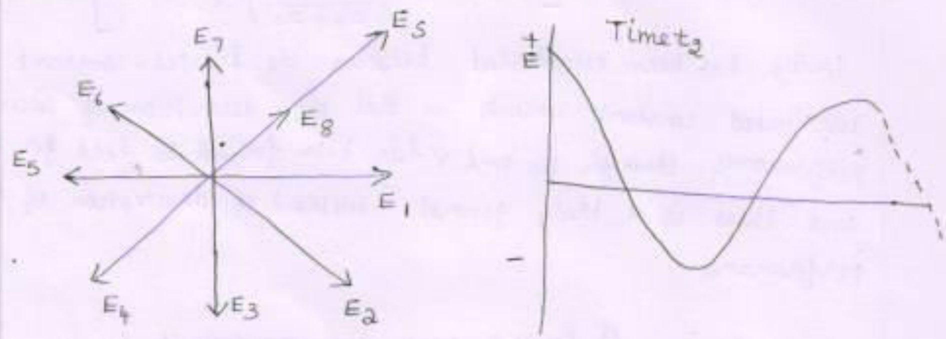
$$E = E_S\,\varepsilon^{-\alpha s}\varepsilon^{-j\beta s} \quad ; \quad I = I_S\,\varepsilon^{-\alpha s}\varepsilon^{-j\beta s}$$

a) Voltage phasors at $\frac{1}{8}$ - wavelength points along a line at a Particular time instant

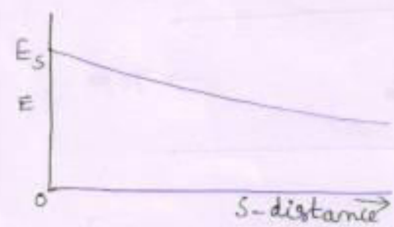
b) A Plot of the phasors:-



c) Voltage phasors one eighth of a cycle later than (a). (d) Plot of voltage along the line for the time instant of (c) showing the movement of the wave along the line



Voltage along an infinite line as measured by an effective reading meter



$$E = E_S \varepsilon^{-\alpha S}$$

$$I = I_S \varepsilon^{-\alpha S}$$

WAVELENGTH, VELOCITY, PROPAGATION:

The distance the wave travels along the line while the phase angle is changing through 2π radians is called a wavelength.

$$\beta\lambda = 2\pi; \quad \lambda = \frac{2\pi}{\beta}$$

$$x = v t; \quad \lambda = \frac{v}{f}; \quad v = \lambda f = \frac{2\pi}{\beta} f; \quad v = \frac{\omega}{\beta}$$

Velocity of propagation is measured in miles per second if β is in radians per mile or in meters per second if β is in radians per meter.

$$Z = R + j\omega L; \quad Y = G + j\omega C; \quad \gamma = \alpha + j\beta = \sqrt{ZY}$$

$$\gamma = \sqrt{RG - \omega^2 LC + j\omega(LG + CR)}$$

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Squaring on both sides,

$$\alpha^2 + j2\alpha\beta - \beta^2 = RG - \omega^2 LC + j\omega(LG + CR)$$

Equating on the real part,

$$\alpha^2 = \beta^2 + RG - \omega^2 LC$$

Equating the imaginary part,

$$4\alpha^2\beta^2 = \omega^2(LG+CR)^2$$

$$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(LG+CR)^2 = 0$$

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+CR)^2}}{2}}$$

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+CR)^2}}{2}}$$

$$\beta = \omega\sqrt{LC}$$

velocity of Propagation $V = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ m/Sec

$$LC = \frac{\pi \varepsilon}{\ln(d/a)}\left(\frac{\mu}{4\pi} + \frac{\mu_v}{\pi}\ln\frac{d}{a}\right)$$

$$LC = \frac{\mu_v \varepsilon_v}{4\ln(d/a)} + \mu_v\varepsilon_v; \quad V = \frac{1}{\sqrt{\mu_v\varepsilon_v\left[\frac{1}{4\ln(d/a)} + 1\right]}} \text{ m/sec}$$

$$V = \frac{1}{\sqrt{\mu_v \varepsilon_v}} \text{ m/sec}$$

m/sec c → velocity of light in Space = 3×10^8

DISTORTION LINE:

If a line is to have neither frequency nor delay distortion, then α and the velocity of Propagation cannot be functions of frequency.

$V = \frac{\omega}{\beta}$; $\beta \rightarrow$ Direct function of frequency.

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

$$R^2G^2 - 2\omega^2 LCRG + \omega^4 L^2C^2 + \omega^2 L^2G^2 + 2\omega^2 LCRG + \omega^2 C^2 R^2 = (RG + \omega^2 LC)^2$$

$$\omega^2 L^2 G^2 - 2\omega^2 LCRG + \omega^2 C^2 R^2 = 0$$

$$(LG - CR)^2 = 0; \quad LG = CR.$$

$\beta = \omega\sqrt{LC}$; The velocity of propagation is $v = \frac{1}{\sqrt{LC}}$

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

$$(RG + \omega^2 LC)^2$$

$\alpha = \sqrt{RG}$; $LG = CR$, $\frac{L}{C} = \frac{R}{G}$.

THE TELEPHONE CABLE:

In the ordinary telephone cable the wires are insulated with paper and twisted in pairs. This construction results in negligible values of inductance and conductance so that reasonable assumptions in the audio range of frequencies are that

$$Z = R$$

$$Y = j\omega C$$

$$\gamma = \sqrt{RG - \omega^2 LC + j\omega (LG + CR)}$$

$L = G = 0$

$$\gamma = \sqrt{j\omega CR} = \sqrt{\frac{j2\omega CR}{2}}$$

$$\gamma = \alpha + j\beta = (1+j1)\sqrt{\frac{\omega CR}{2}}$$

$$\alpha = \sqrt{\frac{\omega CR}{2}}$$

$$\beta = \sqrt{\frac{\omega CR}{2}}$$

Hence the velocity of propagation is

$$V = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{CR}}$$

Both α and the velocity are functions of frequency, such that the higher frequencies are attenuated more and travel faster than the lower frequencies. Very considerable frequency and delay distortion is the result on telephone cable.

REFLECTION ON A LINE NOT TERMINATED IN Z_0 :

Voltages and currents on the line, with s measured as positive from the receiving end,

$$E = \frac{E_R(Z_R + Z_0)}{2 Z_R}\left[\varepsilon^{\gamma s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma s}\right]$$

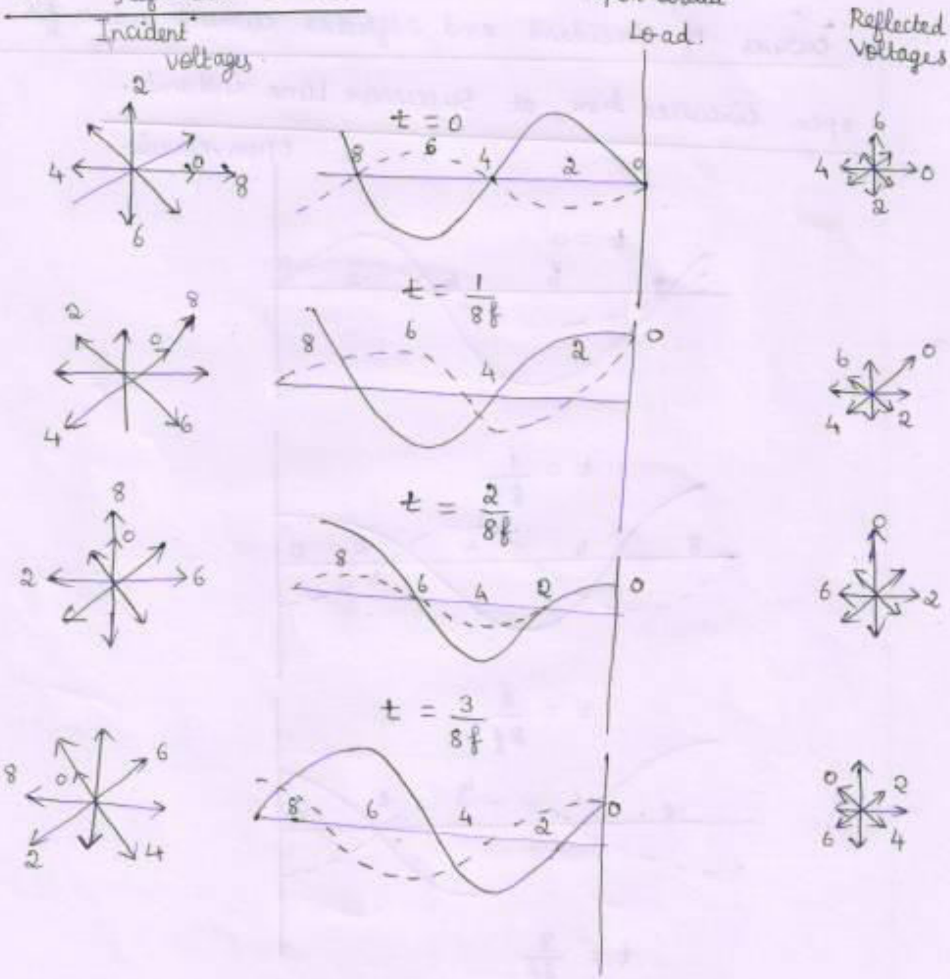
$$I = \frac{I_R(Z_R + Z_0)}{2 Z_0}\left[\varepsilon^{\gamma s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\varepsilon^{-\gamma s}\right]$$

In a direction back along this wave from the receiving end, the amplitude of the wave would increase as s increases, so that the wave that travels from the sending end to the receiving end can be identified as the component varying with $\varepsilon^{\gamma s}$. This wave of voltage or current is known as the incident wave.

In varying with $\varepsilon^{-\gamma s}$, must represent a wave of voltage or current progressing from the receiving end toward the sending end, and decreasing in amplitude with increased distance from the load. (wave). Such a wave of voltage or current is called the reflected wave.

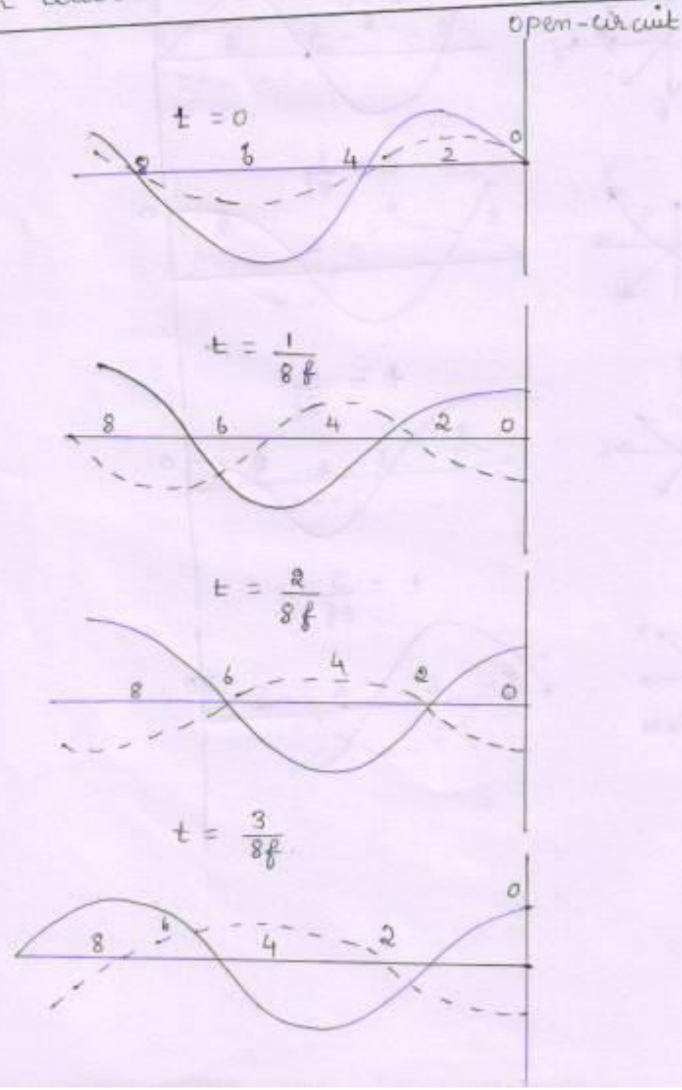
The total instantaneous voltage at any point on the line is the vector sum of the voltage of the incident and reflected waves

Rotating voltage - Phasor Systems for incident and reflected waves for an open circuit termination. The incident wave and reflected waves.



Two current waves are equal and of opposite phase at the open-circuited receiving end, the total instantaneous current at that point always being zero as required by the open circuit.

Curves of incident and reflected current waves for an open circuited line, at successive time instants.



The relative phase angles of the incident and reflected waves at the load are determined by the term $(Z_R - Z_0)/(Z_R + Z_0)$. Therefore both angle and magnitude of Z_R and Z_0 enter into the determination of the phase angle between the two waves.

For an ideal line with $R = G = 0$ and so terminated, the ratio of voltage to current is a constant given by

$$Z_0 = \frac{E}{I}$$

The energy conveyed in the electric field is

$$W_e = \frac{CE^2}{2}$$ joules/m^3 and that conveyed in the magnetic field is

$$W_m = \frac{LI^2}{2}$$ Joules/m^3.

For Such an ideal line $Z_0 = \frac{L}{C}$.

Using this condition for the Z_0 terminated line where $E/I = Z_0$ everywhere, it appears that

$W_e = W_m$ is the physical relation existing everywhere along the ideal line terminated in Z_0.

At a load where $Z_R \neq Z_0$, a different ratio of E_R to I_R.

$Z_R = \frac{E_R}{I_R}$ is required and a redistribution of energy . between the two fields is called for. This redistribution of energy acts as a source to send a reflected wave back along the line.

If the impedance of the generator is not Z_0, the reflected wave is reflected wave is reflected again at the sending end, becoming a new incident wave.

Energy is thus transmitted back and forth on the line until dissipated in the line losses. Hence a termination in Z_0 with no reflection is desirable.

REFLECTION COEFFICIENT:

The ratio of amplitudes of the reflected and incident voltage waves at the receiving end of the line is frequently called the reflection coefficient. with $s = 0$,

This ratio is

$$K = \frac{\text{reflected voltage at load}}{\text{incident voltage at load}}$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$E = \frac{E_R (Z_R + Z_0)}{2 Z_R} \left(\varepsilon^{\gamma s} + K \varepsilon^{-\gamma s} \right)$$

$$I = \frac{I_R (Z_R + Z_0)}{2 Z_0} \left(\varepsilon^{\gamma s} - K \varepsilon^{-\gamma s} \right)$$

The sign of K, and hence the polarity of the reflected wave, is dependent on the angles and magnitudes of Z_R and Z_0. For a termination of Z_0 the reflection coefficient is zero.

The reflection coefficient will be found to be extremely useful.

OPEN AND SHORT CIRCUITED LINES:

An limiting cases it is convenient to consider lines terminated in open circuit or short circuit that is with $Z_R = \infty$ or $Z_R = 0$. The input impedance of a line of length l is

$$Z_s = Z_0 \left(\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right)$$

and for the short-circuit case $Z_R = 0$, so that

$$Z_{sc} = Z_0 \tanh \gamma l.$$

Before the open circuit case is considered, the input impedance should be written

$$Z_s = Z_0 \left[\frac{\cosh \gamma l + (Z_0/Z_R) \sinh \gamma l}{(Z_0/Z_R) \cosh \gamma l + \sinh \gamma l.} \right]$$

The input impedance of the open-circuited line of length l, with $Z_R = \infty$, is

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$Z_{oc'} = Z_0 \cot h \gamma l$

$$Z_0 = \sqrt{Z_{oc} Z_{sc}}$$

This is the Same result as was obtained for a lumped network. Equation Supplies a very valuable means of experimentally determining the value of Z_0 of a line

$$\tan h \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\gamma l = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\tan h \gamma l = \tan h (\alpha + j\beta) l = U + jV$$

Then it can be Shown that

$$\tanh 2\alpha l = \frac{2U}{1+U^2+V^2} \quad \text{and}$$

$$\tan 2\beta l = \frac{2V}{1-U^2-V^2}$$

The value of β is uncertain as to quadrant. Its proper value may be Selected if the approximate velocity of propagation is known.

Micro-optoelectromechanical systems (MOEMS) is also a subset of MST and together with MEMS forms the specialized technology fields using miniaturized combinations of optics, electronics and mechanics. Both their microsystems incorporate the use of microelectronics batch processing techniques for their design and fabrication. There are considerable overlaps between fields in terms of their integrating technology and their applications and hence it is extremely difficult to categorise MEMS devices in terms of sensing domain and/or their subset of MST. The real difference between MEMS and MST is that MEMS tends to use semiconductor processes to create a mechanical part. In contrast, the deposition of a material on silicon for example, does not constitute MEMS but is an application of MST.

Transducer

A transducer is a device that transforms one form of signal or energy into another form. The term transducer can therefore be used to include both sensors and actuators and is the most generic and widely used term in MEMS.

Sensor

A sensor is a device that measures information from a surrounding environment and provides an electrical output signal in response to the parameter it measured. Over the years, this information (or phenomenon) has been categorized in terms of the type of energy domains but MEMS devices generally overlap several domains or do not even belong in any one category. These energy domains include:

- Mechanical - force, pressure, velocity, acceleration, position
- Thermal - temperature, entropy, heat, heat flow
- Chemical - concentration, composition, reaction rate
- Radiant - electromagnetic wave intensity, phase, wavelength, polarization reflectance, refractive index, transmittance
- Magnetic - field intensity, flux density, magnetic moment, permeability
- Electrical - voltage, current, charge, resistance, capacitance, polarization [4,5,6,7]

Actuator

An actuator is a device that converts an electrical signal into an action. It can create a force to manipulate itself, other mechanical devices, or the surrounding environment to perform some useful function.

2.3 History

The history of MEMS is useful to illustrate its diversity, challenges and applications. The following list summarizes some of the key MEMS milestones [8].

1950's

1958 Silicon strain gauges commercially available

1959 "There's Plenty of Room at the Bottom" – Richard Feynman gives a milestone presentation at California Institute of Technology. He issues a public challenge by offering $1000 to the first person to create an electrical motor smaller than 1/64th of an inch.

II - unit (Sensors And Actuat...

Introduction to Actuators I

Electrostatic sensors - parallel plate capacitors
Applications - interdigitated finger capacitan...
Comb drive devices - Micro grippers - Micro M...
Thermal sensing and Actuation - Thermal expansi...
Thermal couples - Thermal resistors - Thermal ...
Applictions - Magnetic Actuators - Micro M...
Components. Case studies of MEMS in ma...
actuators.

Electro Static sensors:

* A capacitor is broadly d... as two conductors that can hold opposit...
* It can be used as eithe... sensor (or) an actuator.
* If the distance are relative ... between two conductors change as a result ... applied stimulus.
* The capacitance value would b... change.
* This form the basis of capa... (or) electrostatic) sensing.
* If a voltage were applied ac... two conductors and electrostatic force would ... develop between these two objects. This is cal...

* Electro Static forces are not often used for driving Macroscopic machinery.

** The electro statically driven Micro Motor was one of the earliest MEMS Actuators.

* The Motor, Consits of a rotor that is attached to the substrate with a hub and a set of fixed electrodes on the periphery. Called Stators.

* The Stators are grouped together such that each group of four electrodes is electrically biased simultaneously

* Simplisity. The sensing and actuation Principles are relatively easy to impliment, requiring only two conducting surfaces.

* Low power Electrostatic actuation relies on differential voltage rather than current.

* The Method is generally Considered energy efficient for low frequency application.

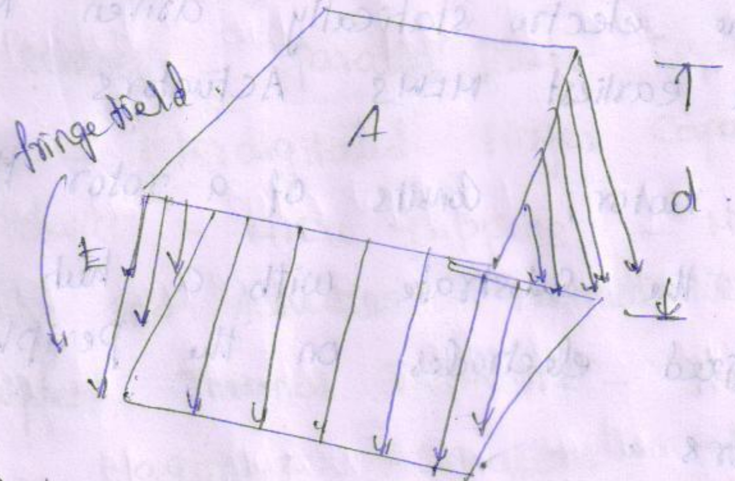
* Fast Response, Low Power.

Parallel Plate capacitor:

Capacitance of ||el plates.

A ||el Plate capacitor is the most fundamental Configuration of electrostatic sensors and Actuators.

* It consists of two conducting plates with their broad sides ||el to each other.



A representative application using ||e plate -Static actuation principle is the DMD chip made by Texas instruments

* A DMD chip consists of a array of Micro mirrors (or) binary light swit

* Each mirror is made of a reflective plate capable of rotating about a torsional support hinge.

* Two electrodes are embedded the reflective plate each covering one half the plate area on each side of the tors hinge.

* Let us examine a simple ||e p capacitor depicted Fig 1. These two plates h an overlapping area of A and a spacing of The dielectric constant (or) relative electrical per of the media between the two plates is den

$$\boxed{E = \frac{Q}{\varepsilon A}}$$

$$C = \frac{Q}{V} = \frac{Q}{E.d} = \frac{Q}{\frac{Q}{\varepsilon A}.d} = \frac{\varepsilon A}{d}$$

$$F = \left|\frac{\partial u}{\partial x}\right| = \frac{1}{2}\left|\frac{\partial C}{\partial x}\right| V^2$$

$$F = \left|\frac{\partial u}{\partial d}\right| = \frac{1}{2}\frac{\varepsilon A}{d^2} V^2 = \frac{1}{2}\frac{CV^2}{d}$$

Applications:

Parallel - plate capacitors can be broadly applied in a variety of sensing and actuation applications

* Parallel plate capacitive accelerometer.
* Torsional ||el plate " "
* Membrane ||el plate sensor.
* " capacitive condenser microphone.
* Inter digit finger capacitance.

Inter digit finger capacitance:

* Alternate fabrication and operation modes compared with ||el plate capacitors.

* Interdigited fingers (IDT) to increase the edge coupling length.

* Two sets of electrodes are placed in the same plane ||el to the substrate.

* Generally one set of finger-like electrode is fixed on chip while a second set is suspended and free to move in one more axes.

* Since the interdigitated fingers are like tooth of combs. Such configuration is referred to the comb drives device.

* Two sets of fingers are in the same

* The distance between a fixed comb and an immediately neighboring movable finger d.

* The thickness of fingers corresponds to of the conductive thin film

* $C = \varepsilon_0 \varepsilon_r \frac{l_0 t}{d}$

$$C_{sl} = C_{sr} = \frac{\varepsilon_0 l_0 t}{x}$$

$$C_{sl} = \frac{\varepsilon_0 l_0 t}{x_0 - x} = \frac{\varepsilon_0 l_0 t}{x_0 + x}$$

$$C_{tot} = C_{sl} + C_{sr} + C_f$$

$$S_e = \frac{\partial C_{tot}}{\partial x}$$

$$F_x = \left|\frac{\partial u}{\partial x}\right| = \left|\frac{\partial}{\partial x}\right| \left(\frac{1}{2} C_{tot}\right.$$

$$C_{sl} = C_{sr} = \frac{\varepsilon_0 (l_0 - y) +}{x_0}$$

$S_y = \frac{\partial C_{rot}}{\partial y}$.

Applications of Comb drives:

* Intertia Sensors.
* Comb drive accelerometer
* ,, ,, Actuator with Large displacement.

Micro grippers and Micro motors:

Micro Motors:

Micro motor is the one of the sensors actuators.

* It consists of a rotor that is attached to the substrate with a hub and a set of fixed electrodes on the periphery called Stators.

* The Stators are grouped together such that each group of four electrodes is electrically biased simultaneously.

* The operation principle of the electrostatic motor, the rotor at an arbitary angular rest position.

* Stator electrode in this group and the closest rotor tooth next to it. This generates an electrostatic attractive force that aligns the said tooth with the said stator electrode.

Thermal Sensing and Actuation :

Thermal Sensors : Classified as :

- Thermal bimorph Sensors.
- Thermal Couples.
- Thermal resistive sensors.

* Temperature sensing is not only useful [illegible] of thermal behaviour.

* The thermal transfer process [illegible] Conduction, Convection or radiation depe[nds on] certain physical variables such as the spa[cing] between objects., travel velocity of media and the properties of materials and me[dia].

Thermal actuators :

* Actuation of microscale d[evices] and structures can be achieved by injecti[ng or] removing heat.

* The temperature of a micro st[ructure] can be raised by absorption of electro magne[tic] ohmic heating, conduction and convection heatin[g].

* Many inkjet printers today [eject] ink droplets using thermal expansion of [the] ~~[illegible]~~ inks.

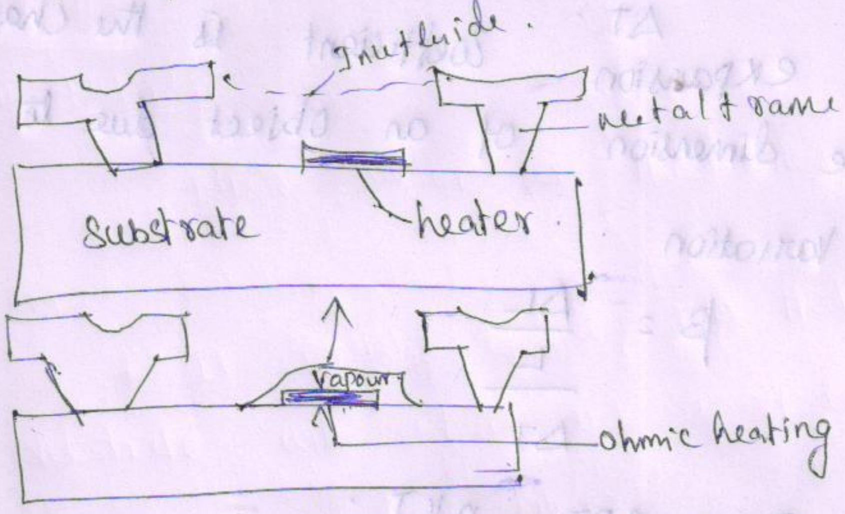
* The volume of ink and the mass associated with the ink or smal[l]

* The heating and cooling of the fluid can be performed at high speed.

* Many commercial ink cartridges, it takes 1 μs or less for the vapor bubble to initiate and 15 μs for the ink to be ejected. The refill of the cavity takes approximately 24 μs.

* The peak temperature at the surface around 330°C.

Principle of ink injection.



There are four possible mechanism for heat to move from one point to another.

(i) Thermal Conduction
(ii) Natural.
(iii) Forced thermal convection.
(iv) Radiations.

Thermal expansions:-

* Thermal expansion is an omni present behaviour of materials.

* The dimensions and volume of structures made of semiconductors, metals and dielectric materials would increase upon temperature.

* The volumetric thermal expansion coefficient (TCE) commonly denoted as α

$$\alpha = \frac{\frac{\Delta V}{V}}{\Delta T}.$$

The linear expansion coefficient is the ch[ange] of only one dimension of an object due to temperature variation

$$\beta = \frac{\frac{\Delta L}{L}}{\Delta T}.$$

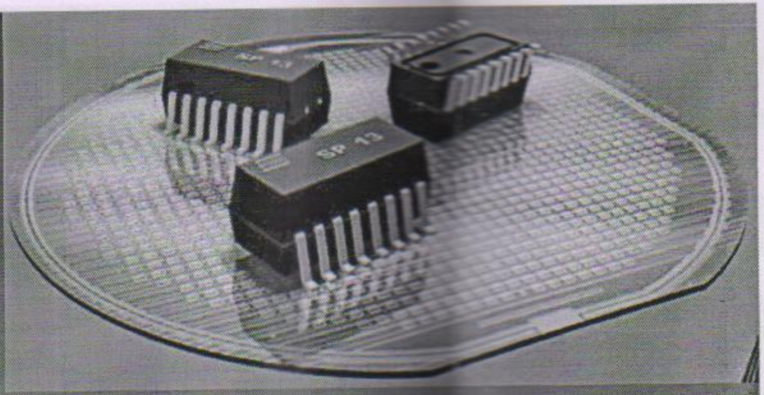
$$PV = nRT = nKT.$$

$$k = R/N_A$$

$$N_A = 6.0221 \times 10^{23}.$$

~~Thermal bimorph Principle:~~

Summary? Unit 2 completed



Microsensors –

15.06.2009

Daniel Lapadatu, SensoNor Technologies
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sensonor

Outline

1. Classification of Sensors
2. Applications
3. MEMS Technologies
4. Pressure Sensors
5. Accelerometers
6. Gyroscopes

Based on:
Adriana Lapadatu, **"Microsensors"**,
1^{st} *e-CUBES Summer School*, Uppsala Univ., 3-5 Sep. 2007,
© 2007 Infineon Technologies.

Classification of Sensors

Mechanical

Force, acceleration, pressure, torque, flow, displacement, velocity, level, position, tilt...

Thermal

Temperature, heat, specific heat, entropy, heat flow...

Chemical

Composition, concentration, reaction rate...

Biosensors

Cells, sugars, proteins, hormones, antigens...

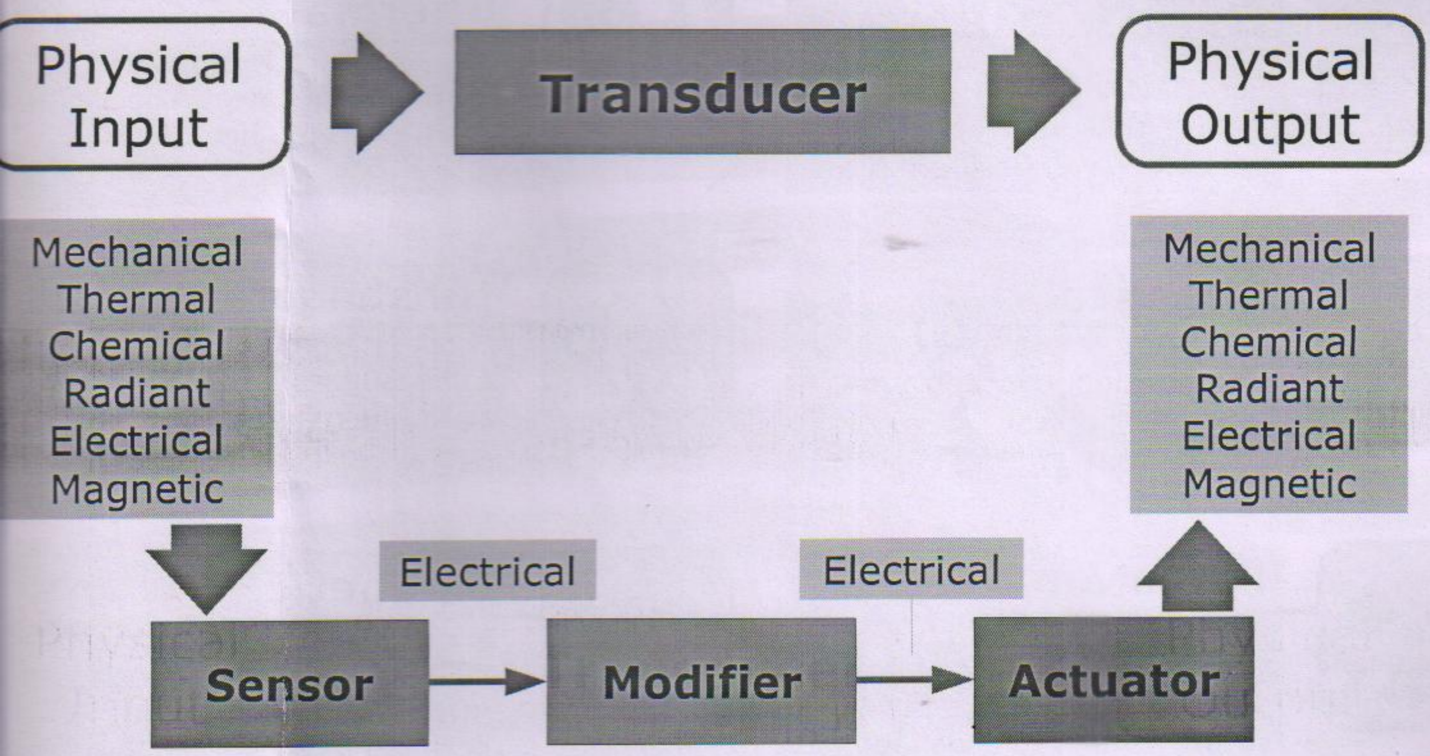
Radiation

Gamma rays, X-rays, ultra-violet, visible light, infrared, microwaves, radio waves...

Magnetic

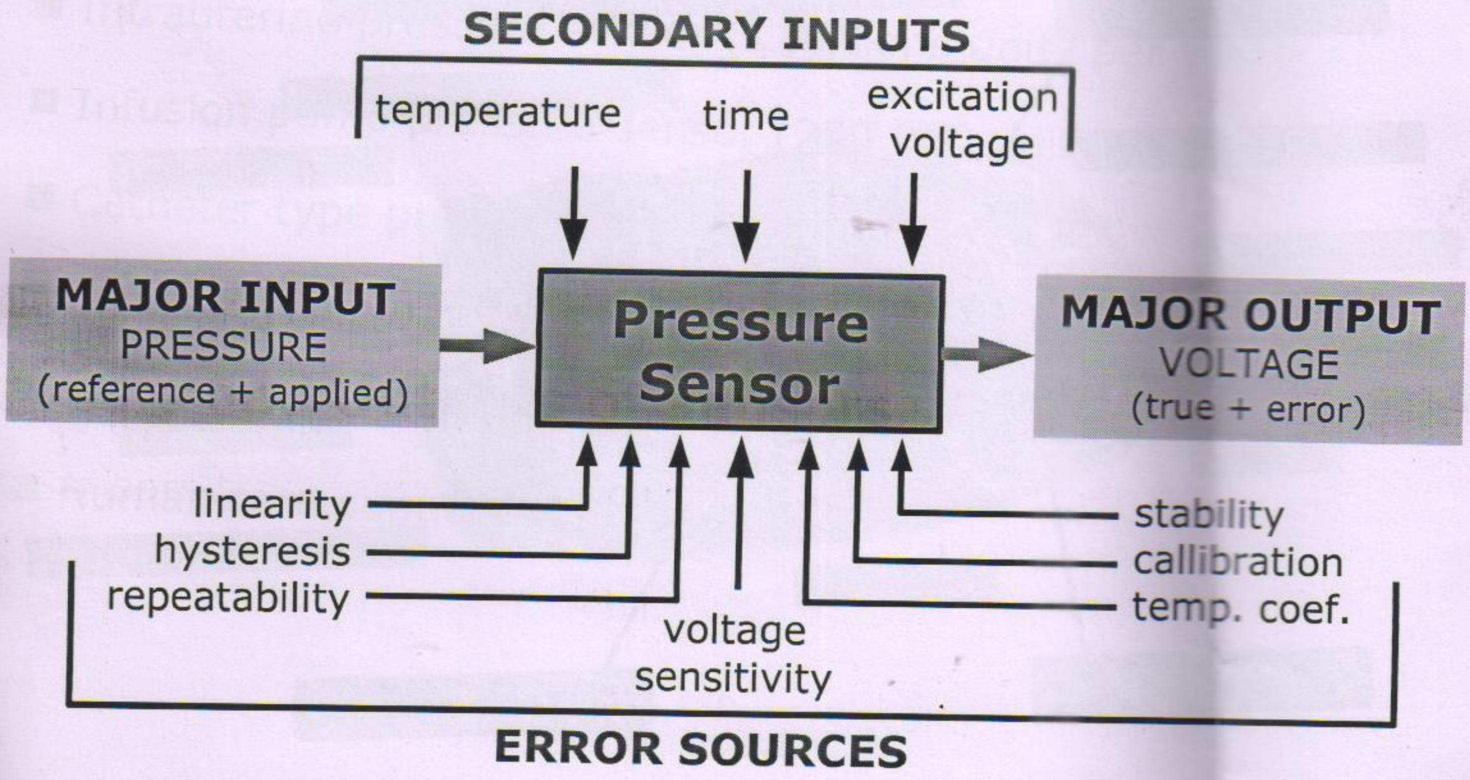
Field intensity, flux density, moment, magnetization, permeability...

ransducers

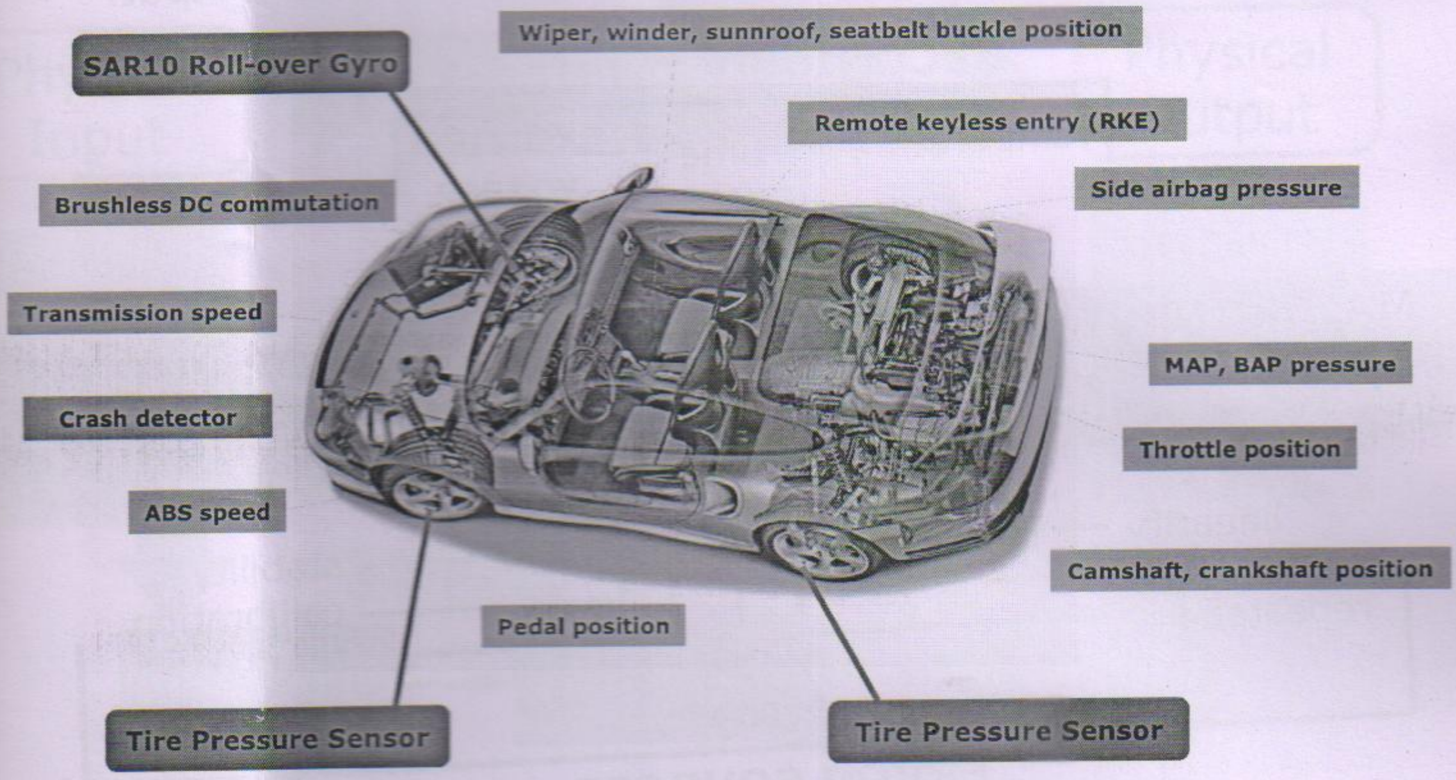

Physical
Input
Transducer
Physical
Output
Mechanical
Thermal
Chemical
Radiant
Electrical
Magnetic
Mechanical
Thermal
Chemical
Radiant
Electrical
Magnetic
Electrical
Electrical
Sensor
Modifier
Actuator

Inputs and Outputs of Sensors

- Sensors transform a *physical* input into an *electrical* output.



Automotive Applications



Medical Applications

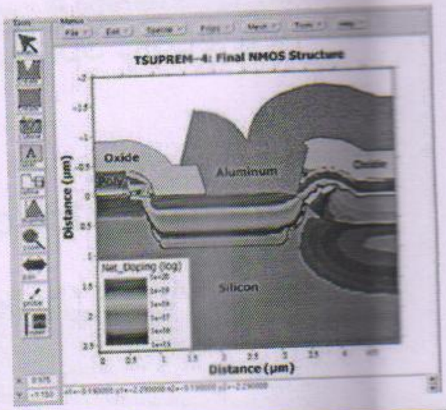
- Disposable blood pressure sensors (17 millions units per year);
- Intrauterine pressure sensor (1 millions units per year);
- Infusion pump pressure sensor (200 000 units per year);
- Catheter type pressure sensors;
- Lungs capacity meters;
- Kidney dialysis equipment;
- Human care support systems...

Examples of Automotive Applications

- **Safety devices:**
  - Airbag deployment,
  - Antilock braking systems (position sensors),
  - Suspension systems (displacement, position and pressure sensors),
  - Object avoidance (pressure and displacement sensors),
  - Navigation (gyroscope).
- **Engine and power train:**
  - Manifold control with pressure sensors,
  - Airflow control,
  - Exhaust gas analysis and control,
  - Fuel pump pressure and fuel injection control,
  - Transmission force and pressure control.
- **Vehicle diagnostics and health monitoring:**
  - Engine oil pressure,
  - Tire pressure,
  - Brake oil pressure,
  - Transmission fluid,
  - Fuel pressure.
- **Comfort:**
  - Seat control,
  - Air quality and flow,
  - Air temperature and humidity,
  - Satellite navigation.

Some Tools

- ***Device design:***
  Cadence, LEdit, Spice, MATLAB, ...



- ***Process design:***
  TSuprem (fabrication cross-section)
  IntelliSuite, AnisE (bulk silicon etching)

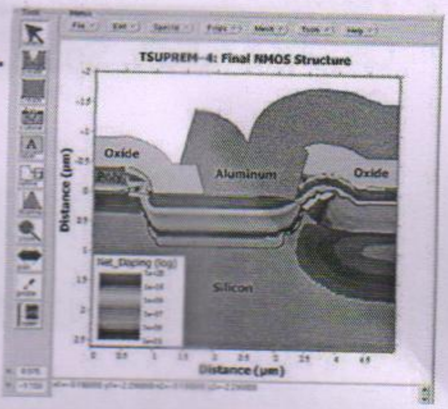
- ***Analysis:***
  FEM systems, analytic tools, MEMCAD, IntelliSuite, ANSYS

Why Use Silicon?

- Take advantage of extensive experience from IC production
- Readily available in very pure form ("nine nines")
- Material properties are very well known
- Can integrate electronics
- Exceptional properties:
  - ➢ ***Very strong***
    Yield strength 7 10^9 N/m^2 vs. steel 4.2 10^9 N/m^2
  - ➢ ***Relatively light***
    Density 2.3 g/cm^3 vs. steel 7.9 g/cm^3
  - ➢ ***Semiconductor***
    Resistivity 0.5 mΩ-cm (doped) to 230 kΩ-cm (intrinsic)

Some Tools

- ***Device design:***
  Cadence, LEdit, Spice, MATLAB, ...



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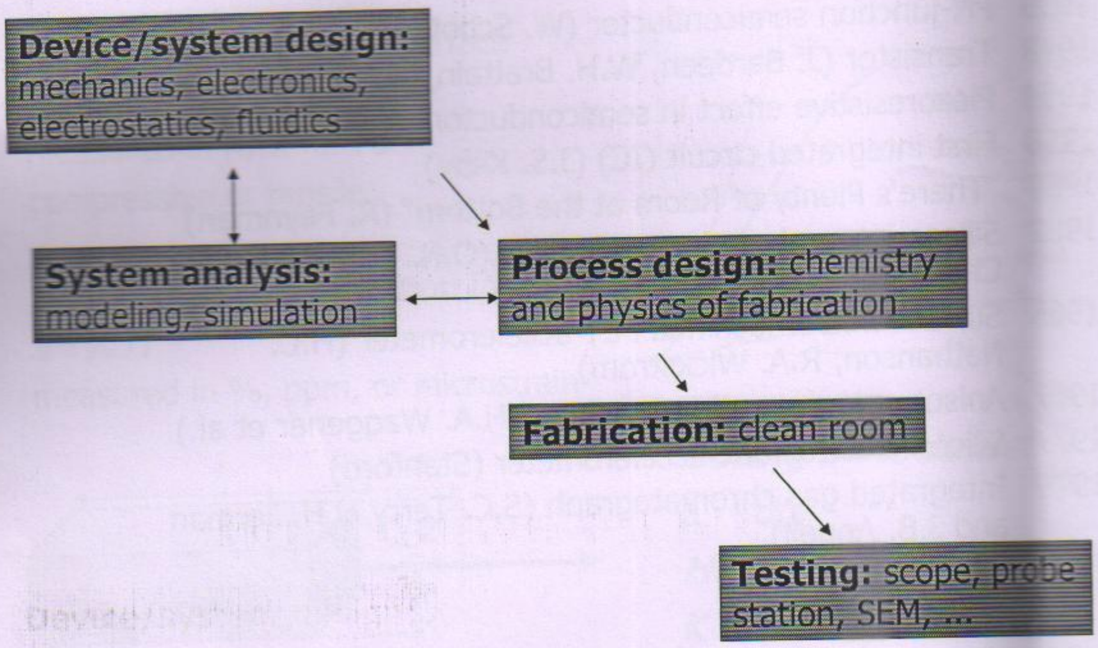
    Resistivity 0.5 mΩ-cm (doped) to 230 kΩ-cm (intrinsic)

How do we do MEMS?



MEMS Design Methodology

- Currently driven by fabrication limitations and processing constraints
- Various specialized CAD tools emerging

Comparison with IC Industry:

- No single standard MEMS process
- No higher-level functional design tools yet (except for very limited applications)

History of MEMS

1939 PN-junction semiconductor (W. Schottky)
1948 Transistor (J. Bardeen, W.H. Brattain, W. Shockley)
1954 Piezoresistive effect in semiconductors (C.S. Smith)
1958 First integrated circuit (IC) (J.S. Kilby)
1959 "There's Plenty of Room at the Bottom" (R. Feynman)
1962 Silicon integrated piezo actuators (O.N. Tufte, P.W. Chapman and D. Long)
1965 Surface micromachined FET accelerometer (H.C. Nathanson, R.A. Wickstrom)
1967 Anisotropic deep silicon etching (H.A. Waggener et al.)
1977 Silicon electrostatic accelerometer (Stanford)
1979 Integrated gas chromatograph (S.C. Terry, J.H. Jerman and J.B. Angell)

History of MEMS

1982 "Silicon as a Mechanical Material" (K. Petersen)
1983 Integrated pressure sensor (Honeywell)
1985 LIGA (W. Ehrfeld et al.)
1986 Silicon wafer bonding (M. Shimbo)
1988 Batch fabricated pressure sensors via wafer bonding (Nova Sensor)
1992 Bulk micromachining (SCREAM process, Cornell)
1993 Digital mirror display (Texas Instruments)
1994 Commercial surface micromachined accelerometer (Analog Devices)
1999 Optical network switch (Lucent)

[Gerlach, Dötzel 1997]

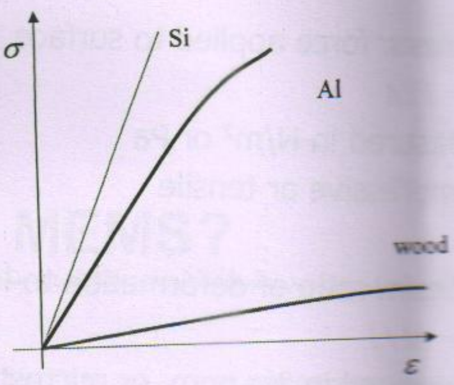
Axial Stress And Strain

Stress: force applied to surface

$\sigma = F/A$

measured in N/m^2 or Pa

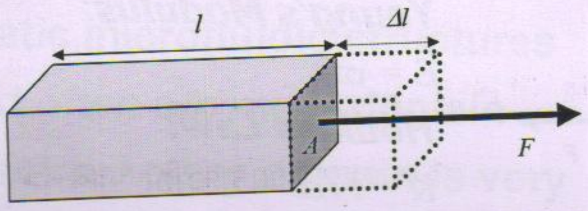
compressive or tensile



Strain: ratio of deformation to length

$\varepsilon = \Delta l / l$

measured in %, ppm, or microstrain



Young's Modulus:

$E = \sigma/\varepsilon$

Hooke's Law:

$K = F/\Delta l = E\,A/l$

Shear Stress And Strain

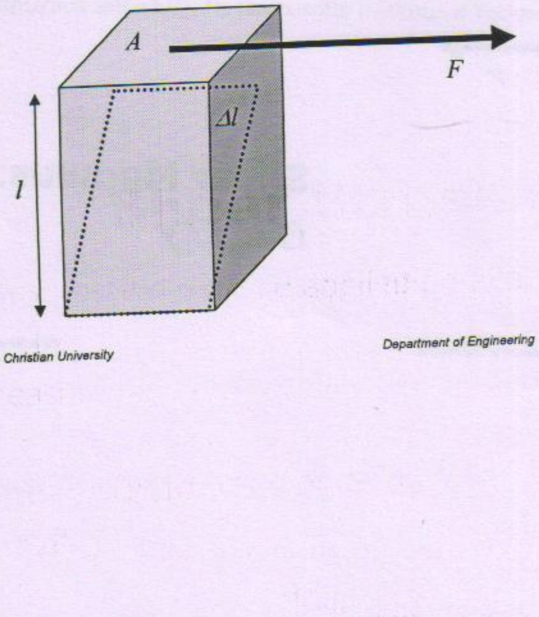
Shear Stress: force applied parallel to surface

$\tau = F/A$

measured in N/m^2 or Pa

Shear Strain: ratio of deformation to length

$\gamma = \Delta l / l$



Shear Modulus:

$G = \tau / \gamma$

Axial Stress And Strain

Stress: force applied to surface

$\sigma = F/A$

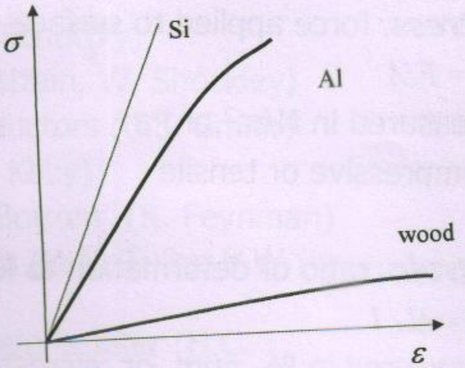
measured in N/m^2 or Pa

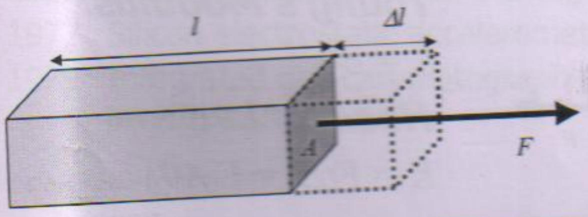
compressive or tensile

Strain: ratio of deformation to length

$\varepsilon = \Delta l / l$

measured in %, ppm, or microstrain





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$K = F/\Delta l = E\,A/l$

Ed Kolesar

Texas Christian University Department of Engineering

Shear Stress And Strain

Shear Stress: force applied parallel to surface

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Shear Strain: ratio of deformation to length

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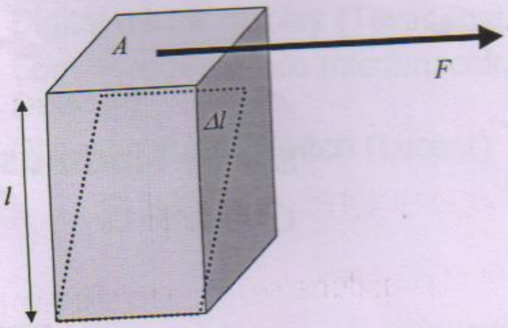


Shear Modulus:

$G = \tau / \gamma$

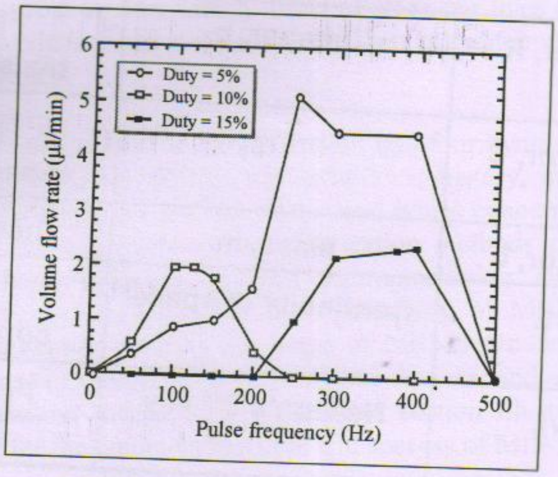
Ed Kolesar

Texas Christian University Department of Engineering

What are MEMS?

> Now, many "MEMS" have no "E" or "M"

- Static microfluidic structures
- But often are multi-domain
- Electro→other domain is very popular
  - » e.g., Electro→Thermal→Fluidic actuation
  - » Microbubble pumps



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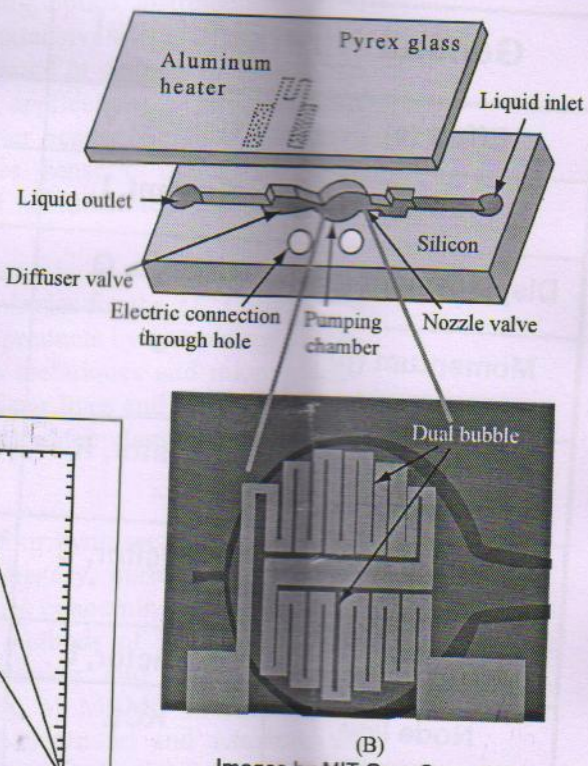


Images by MIT OpenCourseWare.

Liwei Lin (UCB)

Equivalent circuit elements

General	Electrical	Mechanical	Fluidic	Thermal
Effort (e)	Voltage, V	Force, F	Pressure, P	Temp. diff., ΔT
Flow (f)	Current, I	Velocity, v	Vol. flow rate, Q	Heat flow, $\dot{Q}$
Displacement (q)	Charge, Q	Displacement, x	Volume, V	Heat, Q
Momentum (p)	-	Momentum, p	Pressure Momentum, Γ	-
Resistance	Resistor, R	Damper, b	Fluidic resistance, R	Thermal resistance, R
Capacitance	Capacitor, C	Spring, k	Fluid capacitance, C	Heat capacity, mcp
Inertance	Inductor, L	Mass, m	Inertance, M	-
Node law	KCL	Continuity of space	Mass conservation	Heat energy conservation
Mesh law	KVL	Newton's 2nd law	Pressure is relative	Temperature is relative

1. Introduction

This report deals with the emerging field of micro-electromechanical systems, or MEMS. MEMS is a process technology used to create tiny integrated devices or systems that combine mechanical and electrical components. They are fabricated using integrated circuit (IC) batch processing techniques and can range in size from a few micrometers to millimetres. These devices (or systems) have the ability to sense, control and actuate on the micro scale, and generate effects on the macro scale.

The interdisciplinary nature of MEMS utilizes design, engineering and manufacturing expertise from a wide and diverse range of technical areas including integrated circuit fabrication technology, mechanical engineering, materials science, electrical engineering, chemistry and chemical engineering, as well as fluid engineering, optics, instrumentation and packaging. The complexity of MEMS is also shown in the extensive range of markets and applications that incorporate MEMS devices. MEMS can be found in systems ranging across automotive, medical, electronic, communication and defence applications. Current MEMS devices include accelerometers for airbag sensors, inkjet printer heads, computer disk drive read/write heads, projection display chips, blood pressure sensors, optical switches, microvalves, biosensors and many other products that are all manufactured and shipped in high commercial volumes.

MEMS has been identified as one of the most promising technologies for the 21st Century and has the potential to revolutionize both industrial and consumer products by combining silicon-based microelectronics with micromachining technology. Its techniques and microsystem-based devices have the potential to dramatically affect of all of our lives and the way we live. If semiconductor microfabrication was seen to be the first micromanufacturing revolution, MEMS is the second revolution.

This report introduces the field of MEMS and is divided into four main sections. In the first section, the reader is introduced to MEMS, its definitions, history, current and potential applications, as well as the state of the MEMS market and issues concerning miniaturization. The second section deals with the fundamental fabrication methods of MEMS including photolithography, bulk micromachining, surface micromachining and high-aspect-ratio micromachining; assembly, system integration and packaging of MEMS devices is also described here. The third section reviews the range of MEMS sensors and actuators, the phenomena that can be sensed or acted upon with MEMS devices, and a brief description of the basic sensing and actuation mechanisms. The final section illustrates the challenges facing the MEMS industry for the commercialisation and success of MEMS.

2. Micro-electromechanical Systems (MEMS)

2.1 What is MEMS?

Micro-electromechanical systems (MEMS) is a process technology used to create tiny integrated devices or systems that combine mechanical and electrical components. They are fabricated using integrated circuit (IC) batch processing techniques and can range in size from a few micrometers to millimetres. These devices (or systems) have the ability to sense, control and actuate on the micro scale, and generate effects on the macro scale.

MEMS, an acronym that originated in the United States, is also referred to as Microsystems Technology (MST) in Europe and Micromachines in Japan. Regardless of terminology, the uniting factor of a MEMS device is in the way it is made. While the device electronics are fabricated using 'computer chip' IC technology, the micromechanical components are fabricated by sophisticated manipulations of silicon and other substrates using micromachining processes. Processes such as bulk and surface micromachining, as well as high-aspect-ratio micromachining (HARM) selectively remove parts of the silicon or add additional structural layers to form the mechanical and electromechanical components. While integrated circuits are designed to exploit the electrical properties of silicon, MEMS takes advantage of either silicon's mechanical properties or both its electrical and mechanical properties.

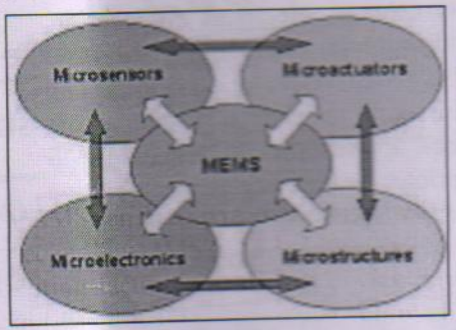


Figure 1. Schematic illustration of MEMS components.

In the most general form, MEMS consist of mechanical microstructures, microsensors, microactuators and microelectronics, all integrated onto the same silicon chip. This is shown schematically in Figure 1.

Microsensors detect changes in the system's environment by measuring mechanical, thermal, magnetic, chemical or electromagnetic information or phenomena. Microelectronics process this information and signal the microactuators to react and create some form of changes to the environment.

MEMS devices are very small; their components are usually microscopic. Levers, gears, pistons, as well as motors and even steam engines have all been fabricated by MEMS (Figure 2). However, MEMS is not just about the miniaturization of mechanical components or making things out of silicon (in fact, the term MEMS is actually misleading as many micromachined devices are not mechanical in any sense). MEMS is a manufacturing technology; a paradigm for designing and creating complex mechanical devices and systems as well as their integrated electronics using batch fabrication techniques.

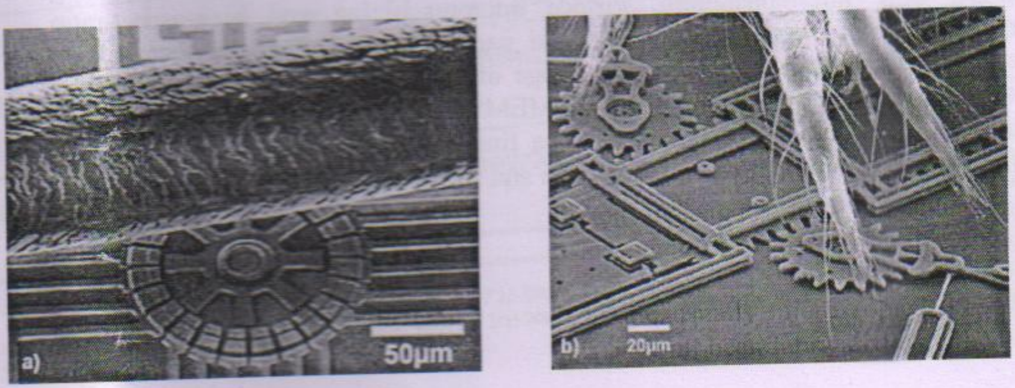


Figure 2. (a) A MEMS silicon motor together with a strand of human hair [1], and (b) the legs of a spider mite standing on gears from a micro-engine [2 - Sandia National Labs, SUMMiT *Technology, http://mems.sandia.gov].

From a very early vision in the early 1950's, MEMS has gradually made its way out of research laboratories and into everyday products. In the mid-1990's, MEMS components began appearing in numerous commercial products and applications including accelerometers used to control airbag deployment in vehicles, pressure sensors for medical applications, and inkjet printer heads. Today, MEMS devices are also found in projection displays and for micropositioners in data storage systems. However, the greatest potential for MEMS devices lies in new applications within telecommunications (optical and wireless), biomedical and process control areas.

MEMS has several distinct advantages as a manufacturing technology. In the first place, the interdisciplinary nature of MEMS technology and its micromachining techniques, as well as its diversity of applications has resulted in an unprecedented range of devices and synergies across previously unrelated fields (for example biology and microelectronics). Secondly, MEMS with its batch fabrication techniques enables components and devices to be manufactured with increased performance and reliability, combined with the obvious advantages of reduced physical size, volume, weight and cost. Thirdly, MEMS provides the basis for the manufacture of products that cannot be made by other methods. These factors make MEMS potentially a far more pervasive technology than integrated circuit microchips. However, there are many challenges and technological obstacles associated with miniaturization that need to be addressed and overcome before MEMS can realize its overwhelming potential.

2.2 Definitions and Classifications

This section defines some of the key terminology and classifications associated with MEMS. It is intended to help the reader and newcomers to the field of micromachining become familiar with some of the more common terms. A more detailed glossary of terms has been included in Appendix A.

Figure 3 illustrates the classifications of microsystems technology (MST). Although MEMS is also referred to as MST, strictly speaking, MEMS is a process technology used to create these tiny mechanical devices or systems, and as a result, it is a subset of MST.

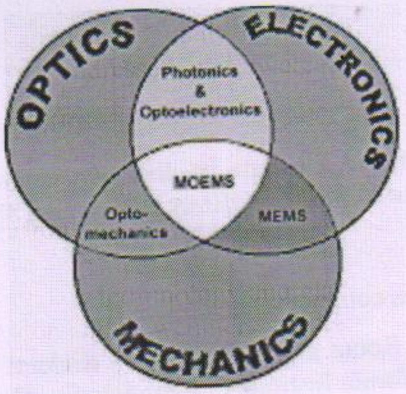


Figure 3. Classifications of microsystems technology [3].

UNIT II

HIGH FREQUENCY TRANSMISSION LINES.

TRANSMISSION LINES.

INTRODUCTION:

For the radio frequency line working at a frequency of the range of megahertz. For the radio frequency line of either open wire type or coaxial line type. At very high frequency, the Skin effect is considerable. Hence it is assumed that the currents may flow on the Surface of conductor. Then the internal inductance becomes zero. The line at radio frequency is constructed such that the leakage conductance G may be considered zero.

PARAMETERS OF OPEN WIRE LINE AT RADIO FREQUENCIES:

For the inductance of an open wire line become.

$$L = \frac{\mu_0}{2\pi} \ln\frac{d}{a} = 4\times10^{-7} \ln\frac{d}{a} \text{ henrys/m} = 9.21\times10^{-7} \log\frac{d}{a} \text{ henrys/m}$$

The value of capacitance of a line is not affected by skin effect or frequency.

For the open-wire line with air dielectric as

$$C = \frac{\pi \varepsilon_v}{\ln\frac{d}{a}} \text{ farads/m} = \frac{27.7}{\ln\frac{d}{a}} \mu\mu f/m$$

$$C = \frac{12.07}{\ln\frac{d}{a}} \mu\mu f/m$$

The effective thickness of the surface layer of current may be considered as $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ meters where μ is the conductor permeability and σ is the conductivity of the conductor material in mhos per meter. For copper, μ is that of space or $\mu = \mu_0 = 4\pi \times 10^{-7}$ and σ has the value 5.75×10^{7} mhos permeter at $20^{\circ}c$. For copper, $\delta = \frac{0.0664}{\sqrt{f}}$.

The resistance of a round conductor of radius a meters to direct current is inversely proportional to the area as $R_{dc} = \frac{k}{\pi a^2}$

$R_{ac} = \frac{k}{2\pi a \delta}$ ⇒ The round conductor with alternating current flowing in a skin of thickness δ is

$$R_{ac} = \frac{k}{2\pi a \delta}$$

The ratio of resistance to alternating current to resistance to direct current is

$$\frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{\pi f \mu \sigma}}{2}$$

For copper,
$$\frac{R_{ac}}{R_{dc}} = 7.53\, a\sqrt{f}$$

$$R_{ac} = \frac{8.33 \times 10^{-8}\sqrt{f}}{a} \text{ ohms/meter of line.}$$

PARAMETERS OF CO-AXIAL CABLE AT RADIO FREQUENCY

This phenomenon eliminates flux linkages due to internal conductor flux, and the inductance of the co-axial line as $L = \frac{\mu_v}{2\pi} \ln\frac{b}{a} = 2 \times 10^{-7} \ln\frac{b}{a}$ henrys/m

$$L = 4.60 \times 10^{-7} \log\frac{b}{a} \text{ henrys/m.}$$

The capacitance of the co-axial line is not affected by frequency, $C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}}$ farads/m; $C = \frac{55.5\,\varepsilon_r}{\ln\frac{b}{a}}$ $\mu\mu f/m$

$$C = \frac{24.14\,\varepsilon_r}{\log\frac{b}{a}} \ \mu\mu f/m.$$

An expression for the resistance of a copper co-axial line obtained,

$$R_{ac} = 4.16 \times 10^{-8}\sqrt{f}\left(\frac{1}{b}+\frac{1}{a}\right) \text{ ohms/m of line}$$

where a and b are outer radius of the inner conductor and inner radius of the outer conductor in meters.

The Shunt susceptance is $y = g + j\omega C$.

$$Pf = \frac{g}{\sqrt{g^2+\omega^2 C^2}} \ ; \ Pf = \frac{g}{\omega C} \ ; \ g = \omega C \times Pf$$

LINE CONSTANTS FOR DISSIPATION:

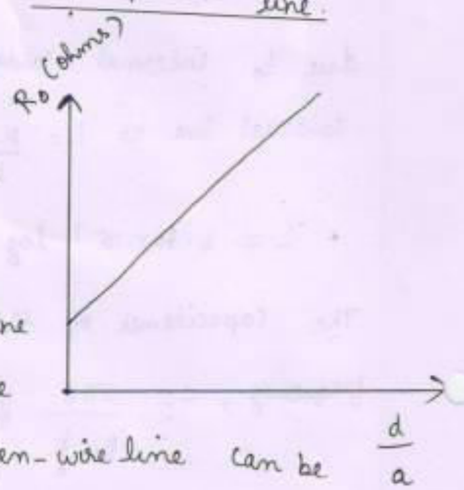
A line of a few wavelengths may be physically short, possibly only a few centimeters.

The Line parameters for the line of zero dissipation are

$$Z = j\omega L$$

$$Y = j\omega C.$$

a) Variation of R_0 with $\frac{d}{a}$ ratio for an open wire line.



The characteristic impedance Z_0 is

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \text{ ohms.}$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}.$$

The capacitance of the open wire line at high frequency, the value of the characteristic impedance of the open-wire line can be found directly from line dimensions as.

$$R_0 = 120 \ln \frac{d}{a} \quad (\text{ohms})$$

$$R_0 = 276 \log \frac{d}{a} \ (\text{ohms})$$

The propagation constant γ is

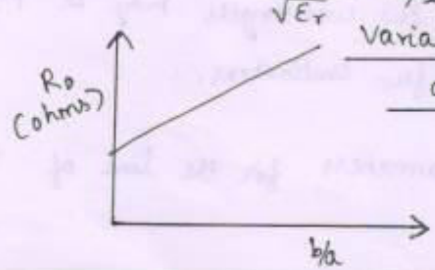
$$\gamma = \sqrt{ZY} = \sqrt{-\omega^2 LC} = \alpha + j\beta = j\omega\sqrt{LC}.$$

$$\alpha = 0, \quad \beta = \omega\sqrt{LC} \text{ radians/m}.$$

The velocity of propagation can be calculated as $V = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ m/sec.

$$V = 3\times10^8 \text{ m/sec}.$$

For the co-axial line $V = \frac{3\times10^8}{\sqrt{\epsilon_r}}$ m/sec.



Variation of R_0 with b/a ratio of a co-axial line.

VOLTAGES AND CURRENTS ON THE DISSIPATION LESSLINE :

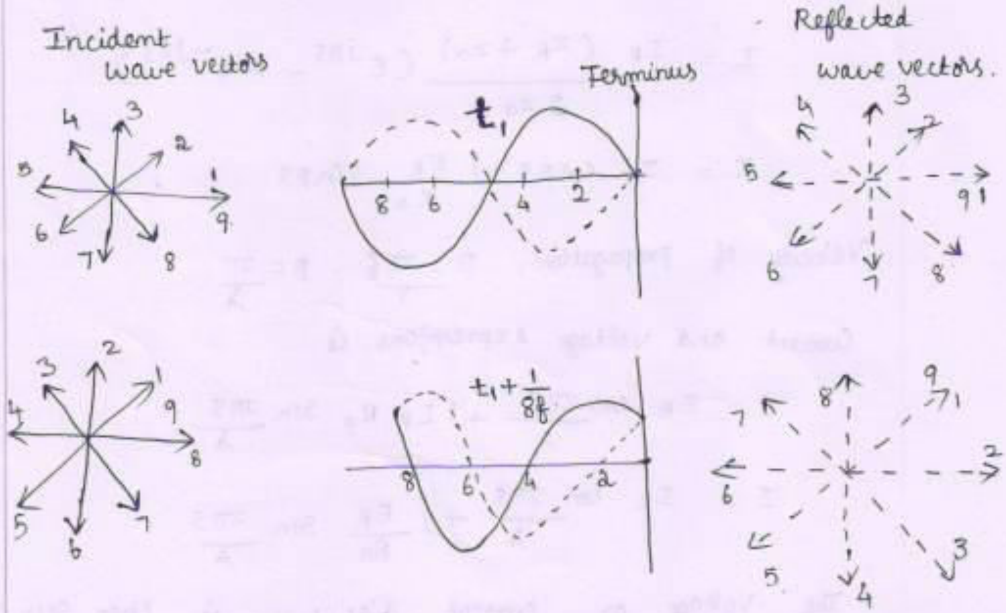
The voltage at any point distant s units from the receiving end of a transmission line is

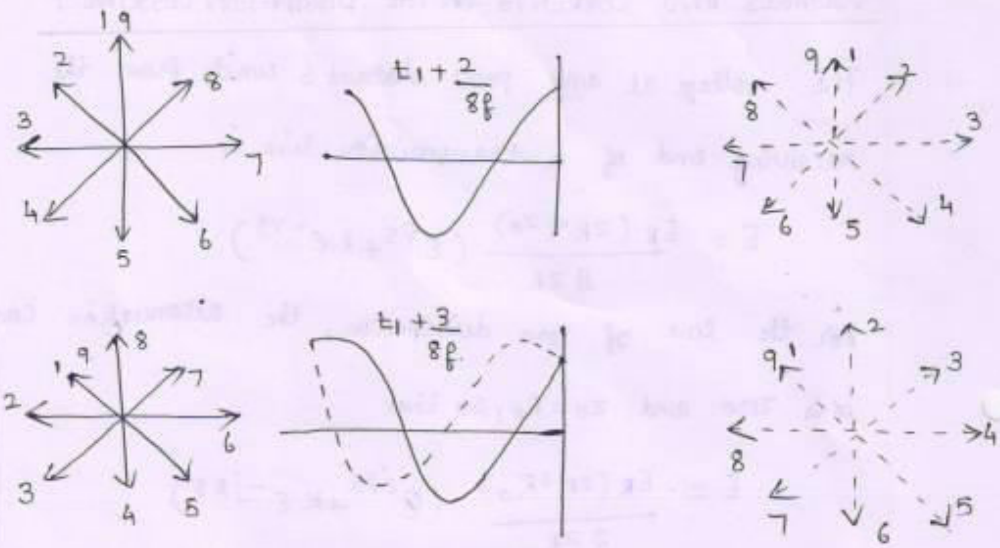
$$E = \frac{E_R (Z_R + Z_0)}{2 Z_R} \left(\varepsilon^{\gamma s} + K \varepsilon^{-\gamma s} \right)$$

For the line of zero dissipation, the attenuation constant α is zero and $Z_0 = R_0$, so that

$$E = \frac{E_R (Z_R + R_0)}{2 Z_R} \left(\varepsilon^{j\beta s} + K \varepsilon^{-j\beta s} \right)$$

Incident and Reflected voltage-wave phasors and values along the direction (dissipationless line) for successive instants of time, for an open-circuited line.





$$E = \frac{E_R}{Z_R}\left[Z_R \frac{(\varepsilon^{j\beta s} + \varepsilon^{-j\beta s})}{2} + j R_0 \frac{(\varepsilon^{j\beta s} - \varepsilon^{-j\beta s})}{2j} \right]$$

$$E = E_R \cos\beta s + j I_R R_0 \sin\beta s.$$

$$I = \frac{I_R (Z_R + Z_0)}{2 Z_0} (\varepsilon^{j\beta s} - K \varepsilon^{-j\beta s})$$

$$I = I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s$$

Velocity of propagation, $\beta = \frac{2\pi f}{v}$; $\beta = \frac{2\pi}{\lambda}$

Current and voltage expressions is

$$E = E_R \cos\frac{2\pi s}{\lambda} + j I_R R_0 \sin\frac{2\pi s}{\lambda}$$

$$I = I_R \cos\frac{2\pi s}{\lambda} + j \frac{E_R}{R_0} \sin\frac{2\pi s}{\lambda}$$

The voltage or current distribution is then seen as the sum of cosine and quadrature sine distributions.

If the line is open circuited, I_R equals zero, and

$$E_{oc} = E_R \cos \frac{2\pi s}{\lambda}$$

$$I_{oc} = \frac{j E_R}{R_0} \sin \frac{2\pi s}{\lambda}$$

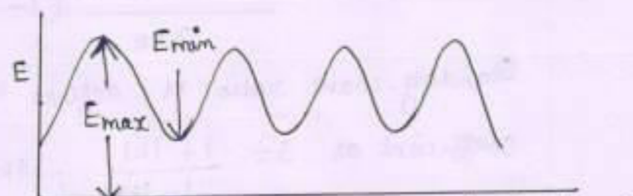
If the line is Short circuited, E_R equals zero and

$$E_{sc} = j I_R R_0 \sin \frac{2\pi s}{\lambda}$$

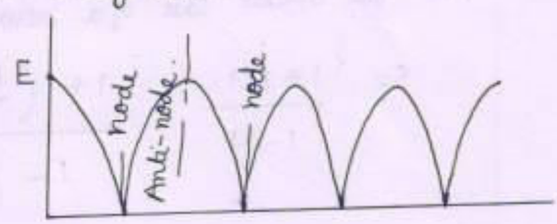
$$I_{sc} = I_R \cos \frac{2\pi s}{\lambda}$$

STANDING WAVES - NODES - STANDING WAVE RATIO:

Standing waves on a dissipationless line terminated in a load not equal to R_0.



Standing waves on a line having open or Short circuit termination.



Nodes are points of zero voltage or current in the Standing wave systems, antinodes or Loops are points of maximum voltage or current.

A line terminated in R_0 has no Standing waves and thus no nodes or Loops and is called a Smooth line.

Standing wave ratio, (S): The ratio of the maximum to minimum magnitudes of current or voltage on a line having standing waves is called the standing-wave ratio, S.

$$S = \left|\frac{E_{max}}{E_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(\varepsilon^{j\beta s} + K\varepsilon^{-j\beta s}\right)$$

$$E_{max} = \frac{E_R(Z_R+Z_0)}{2Z_R}(1+|K|)$$

$$E_{min} = \frac{E_R(Z_R+Z_0)}{2Z_R}(1-|K|).$$

Standing-wave ratio is defined in terms of the reflection coefficient as $S = \frac{1+|K|}{1-|K|}$; $|K| = \frac{S-1}{S+1} = \frac{|E_{max}|-|E_{min}|}{|E_{max}|+|E_{min}|}$

For the special case of a resistive load,

$$S = \frac{1+|K|}{1-|K|} = \frac{1+\left(\frac{R_R-R_0}{R_R+R_0}\right)}{1-\left(\frac{R_R-R_0}{R_R+R_0}\right)}$$

$$S = \frac{R_R}{R_0} \quad (\text{for } R_R > R_0)$$

$$S = \frac{R_0}{R_R} \quad (\text{for } R_R < R_0).$$

INPUT IMPEDANCE OF OPEN AND SHORT CIRCUITED LINES:

The input impedance of a dissipationless line is

$$Z_S = R_0\left(\frac{Z_R + j R_0 \tan \beta S}{R_0 + j Z_R \tan \beta S}\right).$$

For a Short-circuited line, $Z_R = 0$, so that

$$Z_{SC} = j R_0 \tan \beta S$$

$$\beta = \frac{2\pi}{\lambda}, \quad Z_{SC} = j R_0 \tan \frac{2\pi S}{\lambda} \; ; \quad \frac{Z_{SC}}{R_0} = \frac{X}{R_0}$$

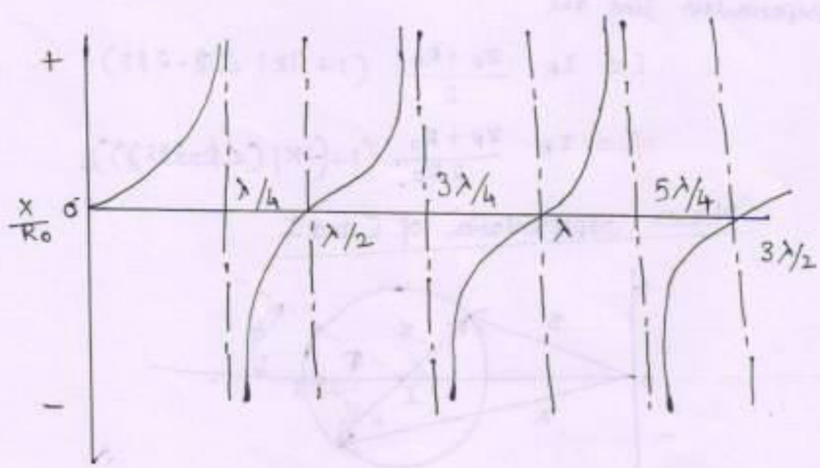
The input impedance of an open-circuited line is determined.

$$Z_S = R_0\left(\frac{1 + j\frac{R_0}{Z_R}\tan\beta S}{\frac{R_0}{Z_R} + j\tan\beta S}\right)$$

For an open-circuited line, $Z_R = \infty$, so that

$$Z_{OC} = \frac{-jR_0}{\tan\beta S} = -j R_0 \cot \frac{2\pi S}{\lambda}$$

SHORT CIRCUIT.



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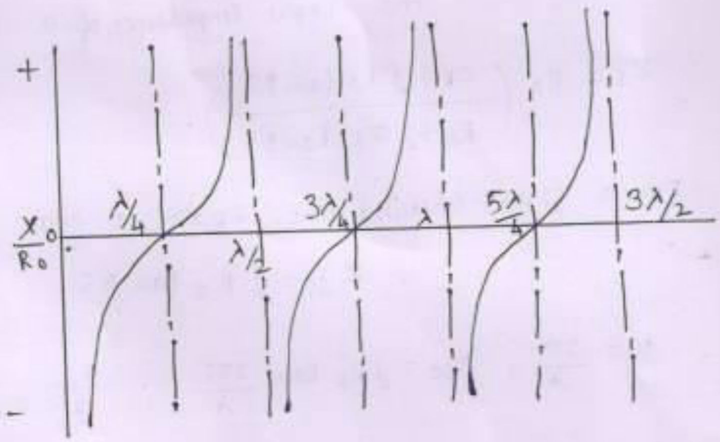
OPEN CIRCUIT



There is a small resistance component of the input impedance indicating some power loss and zero or infinite impedances are never achieved, the actual values tending to minima and maxima.

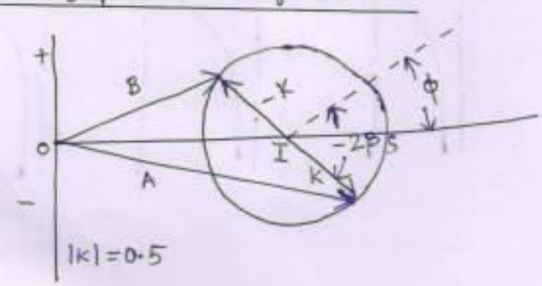
POWER AND IMPEDANCE MEASUREMENT ON LINES

The expressions for voltage and current on the dissipationless line are

$$E = I_R \cdot \frac{Z_R + R_0}{2} \left(1 + |k| \angle \phi - 2\beta s\right)$$

$$I = I_R \cdot \frac{Z_R + R_0}{2R_0} \left(1 + (-)|k| \left(\angle \phi - 2\beta s\right)\right).$$

Phasor representation of E and I



The maximum voltage is $E_{max} = \frac{I_R |Z_R + R_0|}{2}(1+|K|)$

The maximum current is $I_{max} = \frac{I_R |Z_R + R_0|}{2R_0}(1+|K|)$

$$\frac{E_{max}}{I_{max}} = R_0 \quad , \quad \frac{E_{min}}{I_{min}} = R_0 .$$

The expression for minimum value of current is

$$I_{min} = \frac{I_R |Z_R + R_0|}{2R_0}(1-|K|).$$

$$\frac{E_{max}}{I_{min}} = R_0\left(\frac{1+|K|}{1-|K|}\right) = SR_0 \; ; \quad \frac{E_{max}}{I_{min}} = SR_0 = R_{max}$$

$$E_{min} = \frac{I_R |Z_R + R_0|}{2}(1-|K|)$$

$$\frac{E_{min}}{I_{max}} = R_0\left(\frac{1-|K|}{1+|K|}\right) = \frac{R_0}{S}$$

$$\frac{E_{min}}{I_{max}} = R_{min} = \frac{R_0}{S}.$$

The effective power flowing into a resistance R_{max} is the power passing voltage loop at voltage E_{max}.

$$\text{Power } (P) = \frac{E^2_{max}}{R_{max}} \; ; \quad P = \frac{E^2_{min}}{R_{min}}$$

$$P^2 = \frac{E^2_{max}}{R_{max}} \cdot \frac{E^2_{min}}{R_{min}}$$

$$P = \frac{(|E_{max}| \cdot |E_{min}|)}{R_0} \; ; \quad P = (|I_{max}| \cdot |I_{min}|) \cdot R_0$$

The impedance is minimum at a point where voltage is minimum.

$$Z_S = R_{min} = \frac{R_0}{S}.$$

$$Z_S = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi S}{\lambda}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi S}{\lambda}\right)} \right]$$

$$\frac{R_0}{S} = R_{min} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi S'}{\lambda}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi S'}{\lambda}\right)} \right]$$

$$Z_R = R_0 \left[\frac{1 - jS \tan\left(\frac{2\pi S'}{\lambda}\right)}{S + j\tan\left(\frac{2\pi S'}{\lambda}\right)} \right]$$

λ/4 LINE - IMPEDANCE MATCHING:

Expression for the input impedance of the line is

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\beta S)}{R_0 + jZ_R \tan(\beta S)} \right]$$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\beta S)} + jR_0}{\frac{R_0}{\tan(\beta S)} + jZ_R} \right] ; \quad \beta = \frac{2\pi}{\lambda}$$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda} S\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{\lambda} S\right)} + jZ_R} \right]$$

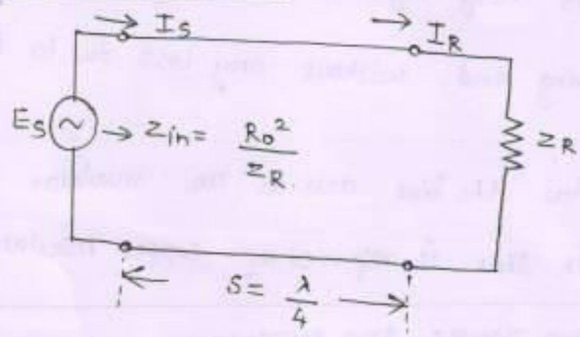
For Quarter-wave line, $S = \frac{\lambda}{4}$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right)} + jZ_R} \right]$$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\frac{\pi}{2})} + j R_0}{\frac{R_0}{\tan(\frac{\pi}{2})} + j Z_R} \right]$$

$$Z_{in} = R_0 \left[\frac{j R_0}{j Z_R} \right] \quad ; \quad Z_{in} = \frac{R_0^2}{Z_R}.$$

The Quarter - wave line

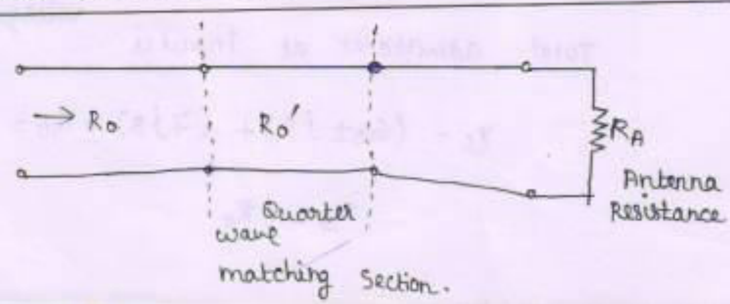


A Quarter wave line may be used as a transformer for impedance matching of load Z_R with input impedance $Z_{in} = Z_R$.

$$R_0 = \left| \sqrt{Z_R \cdot Z_{in}} \right|.$$

A Quarter wave line can transform a low impedance into a high impedance and vice versa, thus it can be considered as an impedance inverter.

Application of Quarter wave line to Couple line to the antenna.



Characteristic impedance of the matching section R_0' is

$$R_0' = \sqrt{R_0\left(\frac{R_0}{S}\right)} = R_0\sqrt{\frac{1}{S}}$$

$$R_0' = \sqrt{R_0(S R_0)} = R_0\sqrt{S}.$$

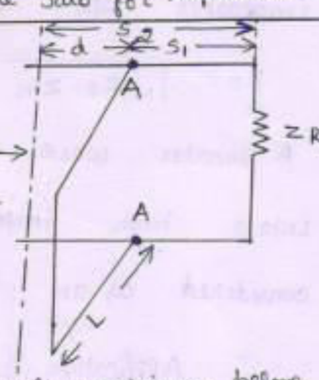
As Quarter wave line is Shorted at ground, its input impedance is very high. So the signal on line passes to the receiving end, without any Loss due to this mechanical support.

Thus the line acts as an insulator at this point. Hence such line is referred as copper insulator.

SINGLE AND DOUBLE-STUB MATCHING

SINGLE-STUB MATCHING:

Location of the Single Stub for impedance matching



when a high frequency line is terminated in its characteristic impedance Y_s → R_0, it is operated as a Smooth line.

Input admittance Y_S,

$$Y_S = G_0 \pm jB.$$

Total admittance at input is

$$Y_S = (G_0 \pm jB) + (\mp jB) = G_0 = \frac{1}{R_0}$$

$$Z_S = R_0$$

Input impedance of the line at point Looking towards Load is

$$Z_S = \frac{1}{Y_S} = R_o$$

The Expression for the input impedance as,

$$Z_{in} = Z_S = R_o \left[\frac{1 + |K| \angle \phi - 2\beta s}{1 - |K| \angle \phi - 2\beta s} \right]$$

Input admittance is

$$Y_S = \frac{1}{Z_S} = \frac{1}{R_o} \left[\frac{1 - |K| \angle \phi - 2\beta s}{1 + |K| \angle \phi - 2\beta s} \right]$$

$$\frac{G_S}{G_o} = S \quad (\text{or}) \quad \frac{R_S}{R_o} = \frac{1}{S}$$

Distance d from voltage minimum to the point of Stub connection is

$$d = \frac{\pm \cos^{-1}\left(\frac{S-1}{S+1}\right)}{\pi} \left(\frac{\lambda}{4}\right).$$

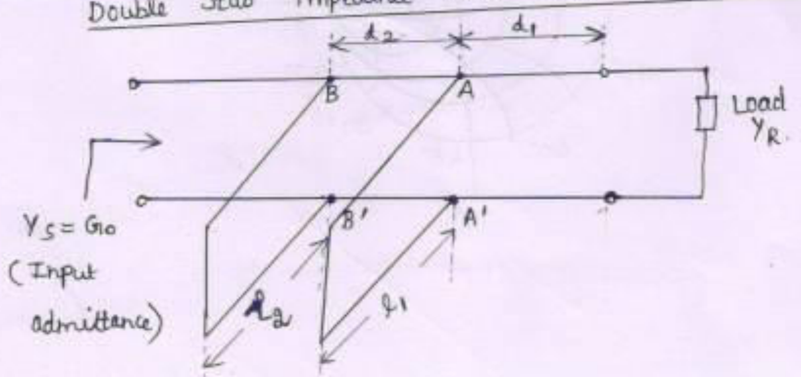
Stub length is given by $L' = \frac{\lambda}{2} - L$

$$L = \frac{\lambda}{2\pi} \tan^{-1} \left[+ \frac{\sqrt{1-|K|^2}}{2|K|} \right] ; \quad S_1 = \frac{\phi + \pi - \cos^{-1}(|K|)}{\pi} \cdot \frac{\lambda}{4}$$

$$L = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{-\sqrt{1-|K|^2}}{2|K|} \right) ; \quad S_1' = \frac{\phi + \pi + \cos^{-1}(|K|)}{\pi} \cdot \frac{\lambda}{4}.$$

DOUBLE STUB MATCHING:

Double Stub impedance to a line with termination Y_R.



Input Impedance of a dissipationless line at any point distance S away from Load is $z_s = z_0 \left[\dfrac{z_R + j z_0 \tan\beta s}{z_0 + j z_R \tan\beta s} \right]$

$$y_s = \frac{y_R + j\tan\beta s}{1 + j y_R \tan\beta s}$$

$$y_s = \frac{y_R (1+\tan^2\beta s)}{1 + y_R^2 \tan^2\beta s} + j \frac{(1 - y_R^2)\tan\beta s}{1 + y_R^2 \tan^2\beta s}$$

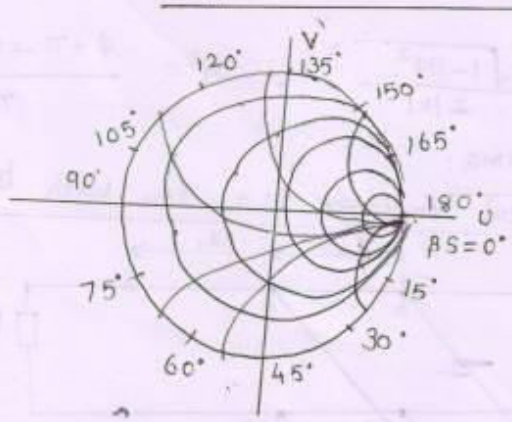
Two Stubs are Seperated by Fixed distances $\frac{\lambda}{16}$, $\frac{\lambda}{8}$, $\frac{3\lambda}{16}$, $\frac{3\lambda}{8}$ etc.....

CIRCLE DIAGRAM, SMITH CHART AND ITS APPLICATIONS:

SMITH CHART - THE SMITH CIRCLE DIAGRAM:

The Smith chart is a Valuable Graphical tool for Solving radio frequency transmission line problems.

SMITH CHART CIRCLE DIAGRAMS:



APPLICATIONS OF THE SMITH CHART:

1. Plotting an Impedance
2. Measurement of VSWR
3. Measurement of Reflection Coefficient K [magnitude and phase]
4. Measurement of Input impedance of the line.
5. Measurement of Impedance to Admittance conversion.

1. Plotting an Impedance:

Normalized impedance $z_R = \frac{Z_R}{R_0} = \frac{60+j40}{50} = 1.2+j0.8$

The intersection of the two circles is represented by the dotted lines and the point P indicates the normalized impedance on the chart.

2. Measurement of VSWR:

Point of intersection of S- circle with the real axis at the right side of the centre indicates a VSWR for given line. - Select a centre of the circle as point O(1,0), Take a distance from O to the point P indicating normalized impedance and then draw a circle: the circle cuts the horizontal real axis. This indicates the value of VSWR.

3. Measurement of Reflection Coefficient K [magnitude and Phase]:

The angle of the reflection coefficient K can be obtained by extending a line from center to the outer rim of the chart through the point which indicates the normalized impedance. The point at which the extended line cuts the outer rim gives directly the value of angle of the reflection coefficient K.

4. Measurement of input impedance of the line:

Calculate Length interms of λ or degrees.

$$\lambda = \frac{v}{f} = \frac{0.6c}{f} = \frac{0.6 \times 3\times10^8}{2\times10^6} = 90\,m$$

$$S = 30m = \frac{30}{90}\lambda = \frac{\lambda}{3} m \text{ or } \frac{720}{3} = 240^\circ$$

Normalized input impedance represented by point T is.

$$z_{in} = 0.48 + j0.035$$

The actual value of the input impedance is

$$Z_{in} = R_0 (z_{in}) = 50\,(0.48 + j0.035) = 24 + j1.75\,\Omega.$$

5. Impedance to Admittance Conversion:

The terminating impedance is.

$$Z_R = 60 + j40$$

Terminating admittance is

$$Y_R = \frac{1}{Z_R} = \frac{1}{60+j40} = \frac{1}{72.111 \angle 33.69} = 0.01386 \angle -33.69^\circ$$

$$= 0.0115 - j7.688 \times 10^{-3}\,℧$$

$$y_R = 0.58 - j0.4.$$

The actual value of the admittance is

$$Y_R = G_0[y_R] = \frac{1}{R_0}[y_R] = \frac{0.58 - j0.4}{50} = 0.016 - j8\times10^{-3}\,℧$$

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Problem Solving Using Smith Chart:

① An R.F. Transmission line with a characteristic impedance of $300 \angle 0^\circ \Omega$ is terminated in an impedance of $100 \angle -45^\circ \Omega$. The load is to be matched to the transmission line by using a short circuited stub. With the help of Smith chart. Determine the length of the stub and the distance from the load.

Solution:

$$Z_R = 100 \angle -45^\circ \Omega = (70.71 - j70.71)\Omega$$

$$z_0 = R_0 = 300 \angle 0^\circ = 300\Omega$$

1. The normalized impedance is

$$z_R = \frac{Z_R}{Z_0} = \frac{Z_R}{Z_0} = \frac{70.71 - j70.71}{300} = 0.2357 - j0.2357$$

2. Locate point A, the intersection of $r = 0.2357$ circle and $x = -0.2357$ circle. As the imaginary component (ie) reactive component of the impedance is negative, the point A is located below the horizontal axis.

3. Draw a circle with O as origin and OA as radius. This is constant S-circle. It cuts real axis. This is the value of S before stub connection.

4. Draw line from point A to O and extend to reach other end on constant S circle. In the chart A, this point is represented as B at which the normalized admittance is $Y_R = 2.1 + j2.1$. Extend line AOB to the outer rim upto point B.

5. Travel along the constant S-circle in the clockwise direction from load to generator to reach a point

UNIT-3 (19)

Quarter wave line & impedance matching: ($\lambda/4$).

The expression for the input impedance of a dissipation less line may be rearranged as.

$$Z_S = R_0 \left[\frac{Z_R + jR_0 \tan \frac{2\pi s}{\lambda}}{R_0 + jZ_R \tan \frac{2\pi s}{\lambda}} \right]$$

$$= R_0 \frac{\tan \frac{2\pi s}{\lambda} \left[\frac{Z_R}{\tan \frac{2\pi s}{\lambda}} + jR_0 \right]}{\tan \frac{2\pi s}{\lambda} \left[\frac{R_0}{\tan \frac{2\pi s}{\lambda}} + jZ_R \right]}$$

In Quarter wave line $S = \lambda/4$

$$Z_S = R_0 \frac{\left[\frac{Z_R}{\tan \frac{2\pi \times \lambda}{\lambda \times 4}} + jR_0 \right]}{\frac{R_0}{\tan \pi/2} + jZ_R}$$

$$= R_0 \times \frac{jR_0}{jZ_R}$$

$$\boxed{Z_S = R_0^2 / Z_R}$$

The i/p impedance of the line is equal to the square of R_0 of the line divided by the load impedance.

Application of Quarter wave section:

(i) A quarter wave section of line may be consider as a transformers, to mach a load of Z_R to a source of Z_S.

(ii) This match can be obtained, if the characteristic impedance R_0' of the matching quarter wave section

i) On line D

$$R_0' = \left| \sqrt{Z_S Z_R} \right| \cdot$$

Impedance inverter:

A quarter wave transformer may be considered as an impedance inverter, it can transform a low impedance into a high impedance and vice versa.

An important application of the quarter wave matching section is to couple a transmission line to a resistive load such as an antenna.

Half wave line:

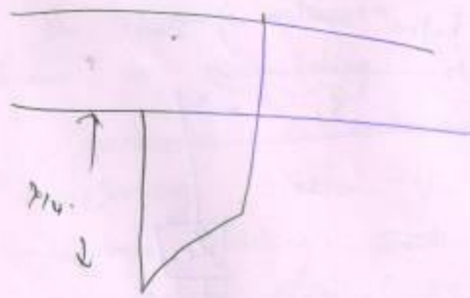
The

$$\boxed{Z_S = Z_R}$$

(21)

A quarter wave section of line may be considered as a transformer to match a load of Z_R to source of Z_s.

$$\boxed{R_0' = \sqrt{Z_s Z_R}}$$



Half wave line:

The input impedance of a dissipationless transmission line is.

$$Z_s = \left[\frac{Z_R + jR_0 \tan \beta s}{R_0 + jZ_R \tan \beta s}\right].$$

For half wave line $s = \lambda/2$.

$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

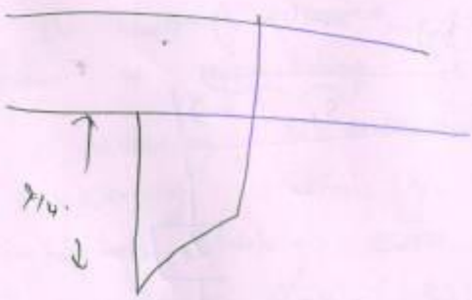
$$Z_s = R_0 \left(\frac{Z_R + jR_0 \tan \pi}{R_0 + jZ_R \tan \pi}\right)$$

$$= R_0 \left(\frac{Z_R}{R_0}\right)$$

$$\boxed{Z_s = Z_R}$$

A quarter wave section of line may be considered as a transformer to match a load of Z_R to source of Z_S.

$$\boxed{R_0' = \sqrt{Z_S Z_R}}$$



Half wave line:

The input impedance of a dissipationless transmission line is

$$Z_S = \left[\frac{Z_R + jR_0 \tan\beta s}{R_0 + jZ_R \tan\beta s}\right]$$

For half wave line $s = \lambda/2$.

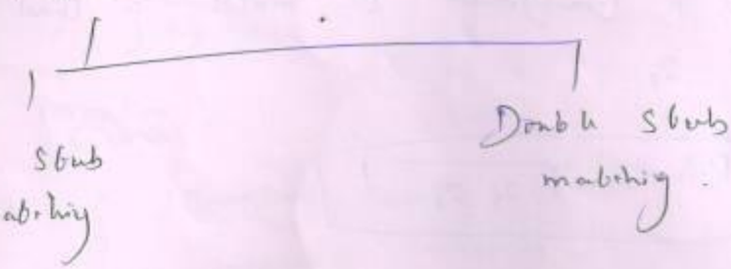
$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z_S = R_0\left(\frac{Z_R + jR_0 \tan\pi}{R_0 + jZ_R \tan\pi}\right)$$

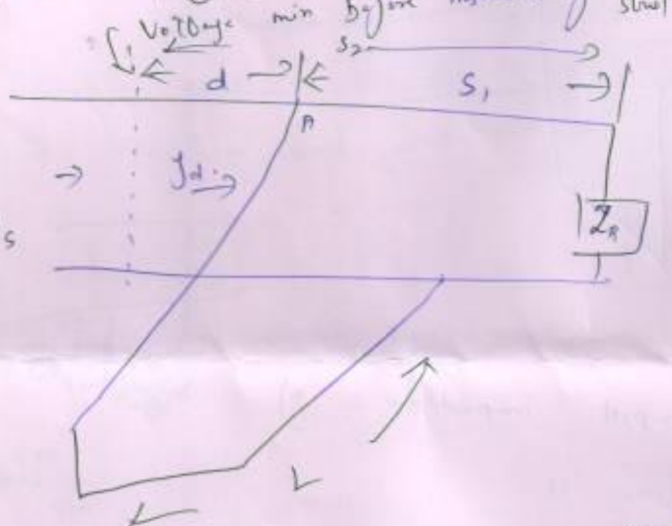
$$= R_0\left(\frac{Z_R}{R_0}\right)$$

$$\boxed{Z_S = Z_R}$$

Stub matching:



Single Stub matching:



The input impedance at any point of a transmission line is given by

$$Z_s = Z_0 \cdot \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}$$

$$Y_s = Y_0 \frac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l}$$

For propagation $\gamma = j\beta$ $(\alpha = 0)$.

$$Y_s = Y_0 \frac{Y_R + j Y_0 \tan \beta l}{Y_0 + j Y_R \tan \beta l}$$

Stub Matching

In general, the source (or) i/p impedance is a fixed one. By selecting the value of load impedance to be equal to the input impedance, impedance matching is achieved.

→ In some cases (Load is an antenna), the load impedance is also fixed.

→ If the load impedance is not equal to the input impedance, the maximum power transfer will not take place. This is known as mismatching.

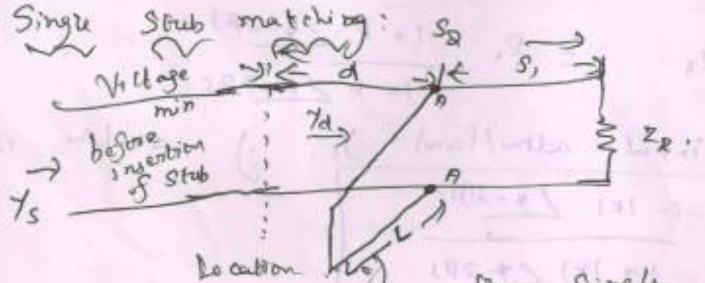
→ So, it is necessary to introduce some form of an impedance transforming section b/w the source & load to achieve impedance matching. ~~Stub~~ This section is called an impedance matching device. (Eg: quarter wave transformer).

→ Another means of achieve impedance matching is the use of an open (or) short circuited line of suitable length called stub. This is called stub matching.

There are two types of stub matching.

1. Single stub matching
2. Double " "

Single Stub matching:



The single stub for impedance matching

The input impedance of the transmission line at point A,

$$\boxed{Z_s = R_o}$$

(12) The line from the Source to A is then terminated in R_0 and is a smooth line.

→ From A to the load, the reflection and standing waves are occur.

→ To avoid reflection.,

→ To select location & length of the stub.

[Voltage minimum]. E.

The ~~input impedance~~, Voltage of a transmission line is

$$E_S = \frac{E_R (Z_R + R_0)}{2 Z_R} \left[e^{j\beta s} + k e^{-j\beta s} \right]$$

$$I_S = \frac{I_R (Z_R + R_0)}{2 R_0} \left(e^{j\beta s} - k e^{-j\beta s} \right).$$

$$Z_S = \frac{E_S}{I_S} = \frac{E_R \cancel{(Z_R + R_0)} / 2 Z_R}{I_R \cancel{(Z_R + R_0)} / 2 R_0} \frac{\left(e^{j\beta s} + k e^{-j\beta s} \right)}{e^{j\beta s} - k e^{-j\beta s}}$$

$$= \frac{E_R}{I_R Z_R} \times R_0.$$

$$= R_0 \frac{\angle \beta s + k \angle \phi - \beta s}{\angle \beta s - k \angle \phi - \beta s.}$$

$$= R_0 \frac{\cancel{\angle \beta s} \left[1 + k \angle \phi - 2\beta s \right]}{\cancel{\angle \beta s} \left[1 - k \angle \phi - 2\beta s \right]}$$

$$Z_S = R_0 \frac{1 + k \angle \phi - 2\beta s}{1 - k \angle \phi - 2\beta s}$$

The input admittance Y_S of a line is

$$\boxed{Y_S = \frac{1}{R_0} \frac{1 - |k| \angle \phi - 2\beta s}{1 + |k| \angle \phi - 2\beta s}}$$

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$G_0 = 1/R_0$.

$$Y_s = G_0 \left[\frac{1 - |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)}{1 + |K| \cos(\phi - 2\beta s) + j|K| \sin(\phi - 2\beta s)} \right]$$

$$= G_0 \left[\frac{1 - |K| \cos(\phi - 2\beta s) - j|K| \sin(\phi - 2\beta s)}{} \right] \cancel{1 + |K| \cos(\phi - 2\beta s)}$$

$$\cancel{= G_0 \left[1 - |K|^2 \right]}$$ — multiply num & denom by.

$$Y_s = G_0 \left[1 + |K| \cancel{\cos(\phi - 2\beta s)} - j|K| \sin(\phi - 2\beta s) - |K| \cancel{\cos(\phi - 2\beta s)} - |K|^2 \cos^2(\phi - 2\beta s) - j|K| \right.$$

$$Y_s = G_0 \left[\frac{1 - |K|^2 - 2j|K| \sin(\phi - 2\beta s)}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)} \right].$$

$$Y_S = G_S + jB_S$$

Conductance Susceptance.

$\phi_m = \beta s_1 + \beta s_2$.

$\phi = G + S$

$$G_s + jB_s = G_0 \left[\quad \right]$$

$$\frac{G_s}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)}.$$

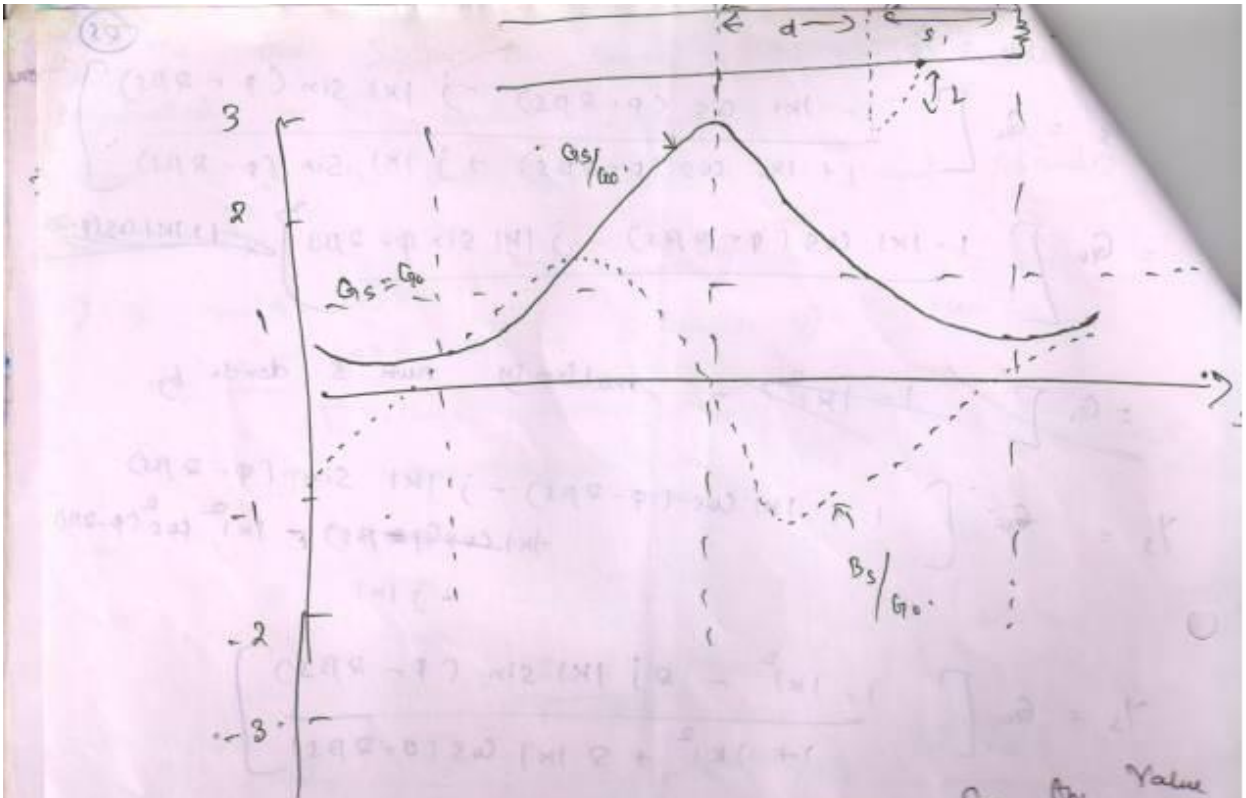
$$\frac{B_s}{G_0} = \frac{-2|K| \sin(\phi - 2\beta s)}{1 + |K|^2 + 2|K| \cos(\phi - 2\beta s)}$$

G_s/G_0 maximum occurs for the value s at which the cosine term is -1.

$$\phi - 2\beta s_2 = -\pi$$

$$2\beta s_2 = \phi + \pi$$

$$\boxed{s_2 = \frac{\phi + \pi}{2\beta}}$$



$|K| = 0.5$; The maximum occurs for the value S_2 at which the cosine term is -1.

$$\phi - 2\beta S_2 = -\pi$$

$$\boxed{S_2 = \frac{\phi + \pi}{2\beta}}$$

At this distance from the load,

$$\frac{G_s}{G_o} = \frac{1 - |K|^2}{1 + |K|^2 - 2|K|}$$

$$= \frac{1+|K|}{1-|K|} = S.$$

$$\frac{1/R_s}{1/R_o} = R_o/R_s = S \qquad \boxed{R_s = R_o/S},$$

In this point of maximum G_s/G_o, is recognized as a point of minimum voltage at a distance S_2 from the load.

In other words, it is used for impedance matching.

The design Parameters of stubs are 1) Stub length 2) Stub distance from the Load.

DESIGN OF SINGLE STUB MATCHING

1) Given Z_L and Z_0 of transmission line, normalize Z_L or find $Z_n = Z_L/Z_0 = r + jx$

2) Mark the Point on the Smith chart where r-circle and x-arc interest

3) Draw a circle with the radius equal to the distance from the centre of the chart to the point.

4) Draw a straight line from this Point through the centre of the chart to the other half of the chart that is travel around it through a distance of $\lambda/4$ to find the Load admittance.

5) Mark the point where the straight line intersects the circle drawn.

6) Note the distance travelled round the circumference of the chart. This is the distance of the stub from the Load.

7) Move clockwise round the Perimeter of the

II - 22

chart and find the point at which the Susceptance tunes out $\pm jb$ Susceptance of the line.

For example if the line admittance is $1+jb$, the required Susceptance is $-jb$.

DOUBLE STUBS

In single stub matching, the stub is placed on the line at a specified point.

Its location varies with Z_L and frequency.

This creates some difficulties as the specified point may occur at an undesirable location.

In such cases, double stubs are used.

The distance between them is fixed such that $\lambda/16$, $\lambda/8$, $3\lambda/16$, $3\lambda/8$

DESIGN OF DOUBLE STUB MATCHING

1) Fix the distance between the two stubs and keep stub 1 at the location of the load.

2) Draw the circle corresponding to the normalised conductance, $g=1$

3) Obtain the normalised distance of d/λ

4) Rotate the circle in anticlockwise direction by d/λ wavelength towards the load

II - (21)
II - (23)

. y_i is located here

5) Locate $y_L = g_L + jb_L$

6) Draw $g = g_L$ circle. This intersects the rotated $g=1$ circle at one or two points where $y_L = g_L + jb_L$.

7) Locate the corresponding y_2 points on the $g=1$ circle $y_2 = 1 + jb_2$

8) Find the Stub Length l_1 between the points representing y_l and y_L

9) Find the Stub length l_2 & from the angle between the point representing $-jb_2$ & P_{sc}

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Input Impedance of a dissipationless line at any point distance S away from load is $z_S = z_0 \left[\frac{z_R + j z_0 \tan\beta s}{z_0 + j z_R \tan\beta s} \right]$

$$Y_S = \frac{Y_R + j\tan\beta s}{1 + jY_R \tan\beta s}$$

$$Y_S = \frac{Y_R(1+\tan^2\beta s)}{1+Y_R^2\tan^2\beta s} + j\frac{(1-Y_R^2)\tan\beta s}{1+Y_R^2\tan^2\beta s}$$

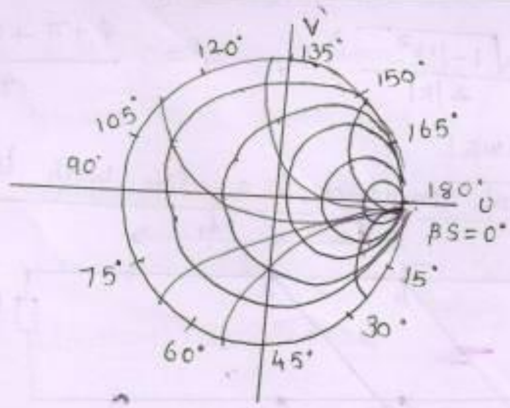
Two stubs are seperated by fixed distances $\frac{\lambda}{16}$, $\frac{\lambda}{8}$, $\frac{3\lambda}{16}$, $\frac{3\lambda}{8}$ etc.....

CIRCLE DIAGRAM, SMITH CHART AND ITS APPLICATIONS:

SMITH CHART - THE SMITH CIRCLE DIAGRAM:

The Smith chart is a valuable graphical tool for solving radio frequency transmission line problems.

SMITH CHART CIRCLE DIAGRAMS:



APPLICATIONS OF THE SMITH CHART:

1. Plotting an Impedance
2. Measurement of VSWR
3. Measurement of Reflection Coefficient K [magnitude and phase]
4. Measurement of Input impedance of the line.
5. Measurement of Impedance to Admittance Conversion.

1. Plotting an Impedance:

Normalized impedance $z_R = \frac{Z_R}{R_0} = \frac{60+j40}{50} = 1.2+j0.8$

The intersection of the two circles is represented by the dotted lines and the point P indicates the normalized impedance on the chart.

2. Measurement of VSWR:

Point of intersection of S-circle with the real axis at the right side of the centre indicates a VSWR for given line. - Select a centre of the circle as point O(1,0), Take a distance from O to the point P indicating normalized impedance and then draw a circle. The circle cuts the horizontal real axis. This indicates the value of VSWR.

3. Measurement of Reflection Coefficient K [magnitude and phase]:

The angle of the reflection coefficient K can be obtained by extending a line from center to the outer rim of the chart through the point which indicates the normalized impedance. The point at which the extended line cuts the outer rim gives directly the value of angle of the reflection coefficient K.

4. Measurement of input impedance of the line:

Calculate Length interms of λ or degrees.

$$\lambda = \frac{v}{f} = \frac{0.6c}{f} = \frac{0.6 \times 3\times10^8}{2\times10^6} = 90\,m.$$

$$S = 30m = \frac{30}{90}\lambda = \frac{\lambda}{3}\,m \text{ or } \frac{720}{3} = 240^\circ$$

Normalized input Impedance represented by point T is.

$$Z_{in} = 0.48 + j0.035$$

The actual value of the input impedance is

$$Z_{in} = R_0 (z_{in}) = 50\,(0.48 + j0.035) = 24 + j1.75\Omega.$$

5. Impedance to Admittance Conversion:

The terminating impedance is.

$$Z_R = 60 + j40$$

Terminating admittance is

$$Y_R = \frac{1}{Z_R} = \frac{1}{60+j40} = \frac{1}{72.111\angle 33.69} = 0.01386\angle-$$

$$= 0.0115 - j7.688\times10^{-3}\ \mho$$

$$y_R = 0.58 - j0.4.$$

The actual value of the admittance is

$$Y_R = G_0[y_R] = \frac{1}{R_0}[y_R] = \frac{0.58 - j0.4}{50} = 0.0$$

(45)

Problem Solving using Smith chart:

① An R.F. Transmission line with a characteristic impedance of $300\ \angle 0^\circ \Omega$ is terminated in an impedance of $100\ \angle -45^\circ \Omega$. The Load is to be matched to the transmission line by using a Short circuited Stub. With the help of Smith Chart. Determine the length of the Stub and the distance from the load.

Solution:

$Z_R = 100\ \angle -45^\circ \Omega = (70.71 - j70.71)\Omega$

$z_0 = R_0 = 300\ \angle 0^\circ = 300\Omega$

1. The normalized impedance is

$$z_R = \frac{Z_R}{Z_0} = \frac{Z_R}{Z_0} = \frac{70.71 - j70.71}{300} = 0.2357 - j0.2357$$

2. Locate point A, the intersection of $r = 0.2357$ circle and $x = -0.2357$ circle. As the imaginary component (ie) reactive component of the impedance is negative, the point A is located below the horizontal axis.

3. Draw a circle with O as origin and OA as radius. This is constant S-circle. It cuts real axis. This is the value of S before Stub connection.

4. Draw line from point A to O and extend to reach other end on constant S circle. In the chart A, this point is represented as B at which the normalized admittance is $Y_R = 2.1 + j2.1$. Extend line AOB to the outer rim upto point B.

5. Travel along the constant S-circle in the clockwise direction from Load to generator to reach a point

UNIT IV PASSIVE FILTERS

INTRODUCTION :

An Electric Filter is a network with the Following Properties.

(i) It can transmit signals within a specified Frequency band. This Frequency band is called Passband.

(ii) It suppresses or attenuates signals outside the Passband. The Frequency over which the Signal is attenuated is called Stop band or attenuation band.

In the case of Passive Filters with L & C network, in the Stop band, attenuation becomes very high and no signal passes to the Load. In the stop band, actually the filter as well as the Load reflect the power back to the source & no power is delivered to the Load through the filter. There will be very little attenuation or consumption of Power by the filter itself.

USES OF FILTERS:

① In Regulated d.c. power supplies, Filters are used to remove the ripples.

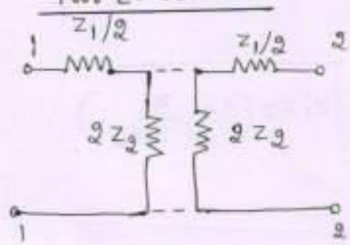
② In radio transmitters, using single sideband operation, Filters are used to eliminate the unwanted side bands.

③ In electrical engineering, Low Pass and high Pass Filters, are used in thyristor controlled circuits to eliminate undesired frequency components.

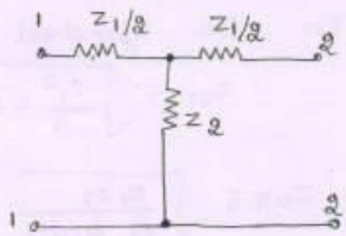
CHARACTERISTIC IMPEDANCE OF SYMMETRICAL NETWORKS:

When $z_1 = z_2$ or the two series arms of a T network are equal or $Z_a = Z_c$ and the shunt arms of a π network are equal, the networks are said to be symmetrical:

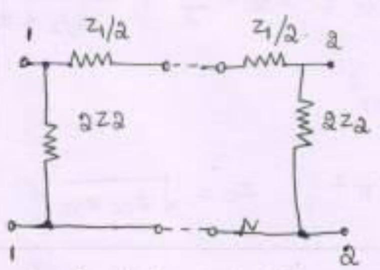
Two L- Sections.



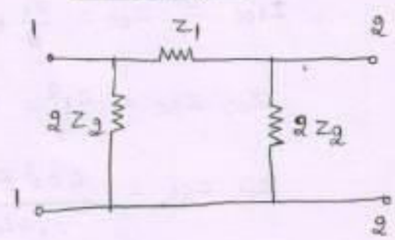
T Section



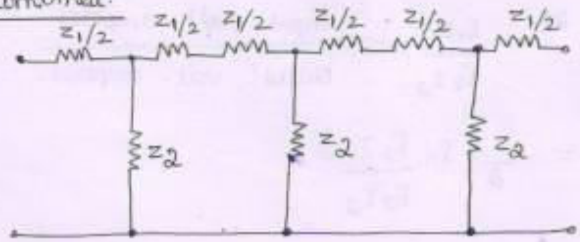
Two L-Sections

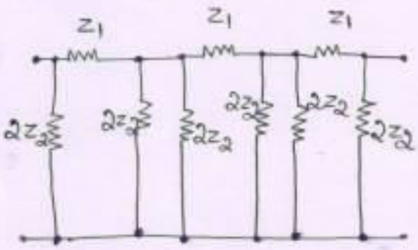


π - Section



a) Ladder network made from T Sections; b) Ladder network built from π sections. The parallel shunt arms will be combined.





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The input impedance is

$$z_{1in} = \frac{z_1}{2} + \frac{z_2\,(z_1/2 + z_0)}{z_1/2 + z_2 + z_0}$$

$$z_0 = \frac{z_1^2/4 + z_1 z_2 + z_2 z_0 + z_1 z_0/2}{z_1/2 + z_2 + z_0}$$

$$z_0^2 = \frac{z_1^2}{4} + z_1 z_2$$

For the Symmetrical T Section,

$$z_{0T} = \sqrt{\frac{z_1^2}{4} + z_1 z_2} = \sqrt{z_1 z_2\left(1 + \frac{z_1}{4z_2}\right)}$$

$$z_{0\pi} = \sqrt{\frac{z_1 z_2}{1 + z_1/4z_2}}$$

$$z_{1oc} = z_{oc} = \frac{z_1}{2} + z_2 \;, \quad z_{1sc} = z_{sc} = \frac{z_1}{2} + \frac{z_1 z_2/2}{z_1/2 + z_2}$$

$$z_{oc}\, z_{sc} = \frac{z_1^2}{4} + z_1 z_2 = z_{0T}^2$$

$$z_{oc}\, z_{sc} = \frac{4 z_2^2 z_1}{z_1 + 4 z_2} = z_{0\pi}^2 \;; \quad z_0 = \sqrt{z_{oc}\, z_{sc}}$$

CURRENT AND VOLTAGE RATIOS- PROPAGATION CONSTANT:

To Define a Transfer constant θ, by

$$\varepsilon^{2\theta} = \frac{E_1 I_1}{E_2 I_2} = \frac{\text{Input volt-amperes}}{\text{output volt-amperes}}$$

$$\theta = \frac{1}{2} \ln \frac{E_1 I_1}{E_2 I_2}$$

where θ is in General a complex number. For symmetrical networks z_0 and the phase angle between the currents.

$\frac{I_1}{I_2} = \varepsilon^{\gamma}$ where γ is a complex number defined as.

$\gamma = \alpha + j\beta$; $\frac{I_1}{I_2} = \varepsilon^{\gamma} = \varepsilon^{\alpha + j\beta}$

$I_1/I_2 = A \angle \beta$; $A = |I_1/I_2| = \varepsilon^{\alpha}$; $\angle \beta = \varepsilon^{j\beta}$

$\frac{V_1}{V_2} = \varepsilon^{\gamma}$.

The term γ (Propagation constant). The Exponent α is known as the attenuation constant.

$$\frac{I_1}{I_2} \times \frac{I_2}{I_3} \times \frac{I_3}{I_4} \times \cdots = \frac{I_1}{I_n}$$

$$\varepsilon^{\gamma_1} \times \varepsilon^{\gamma_2} \times \varepsilon^{\gamma_3} \times \cdots = \varepsilon^{\gamma_n}$$

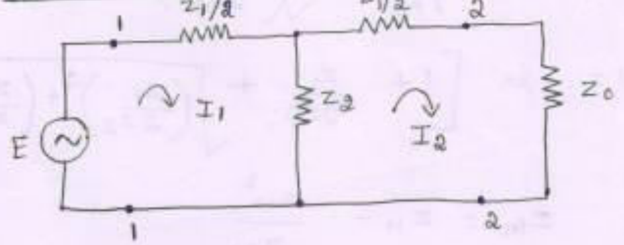
Taking Logarithm

$$\gamma_1 + \gamma_2 + \gamma_3 + \cdots = \gamma_n$$

Thus the overall propagation constant is equal to the sum of the individual propagation constants.

PROPERTIES OF SYMMETRICAL NETWORKS:

Symmetrical Network with Generator and Load:-



The T- Network is considered equivalent to any connected symmetrical network, and is terminated in a load Z_0. The mesh equations are

$$E = I_1\left(\frac{Z_1}{2} + Z_2\right) - I_2 Z_2$$

$$0 = -I_1 Z_2 + I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{Z_1/2 + Z_2 + Z_0}{Z_2} = \varepsilon^{\gamma}$$

$$Z_0 = Z_2(\varepsilon^{\gamma} - 1) - \frac{Z_1}{2}$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$Z_2(\varepsilon^{\gamma} - 1)^2 - Z_1 \varepsilon^{\gamma} = 0, \quad \varepsilon^{2\gamma} - 2\varepsilon^{\gamma} + 1 = \frac{Z_1}{Z_2}\varepsilon^{\gamma}$$

$$\frac{\varepsilon^{\gamma} + \varepsilon^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}; \quad \cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh^2\gamma - \sinh^2\gamma = 1$$

$$\sinh\gamma = \frac{Z_0}{Z_2}; \quad \tanh\gamma = \frac{Z_0}{Z_1/2 + Z_2}$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{1}{2}\left(1 + \frac{Z_1}{2Z_2} - 1\right)} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\varepsilon^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\left(\frac{Z_1}{2Z_2}\right)^2 + \frac{Z_1}{Z_2}}$$

$$\gamma = \ln\left[1 + \frac{Z_1}{2Z_2} + \sqrt{\left(\frac{Z_1}{2Z_2}\right)^2 + \left(\frac{Z_1}{Z_2}\right)}\right]$$

$$Z_{in} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

6.1

$$Z_{in} = \frac{Z_1}{2} + Z_2 - \frac{Z_2^2}{Z_1/2 + Z_2 + Z_R}$$

$$Z_{in} = Z_0 \left(\frac{Z_R \cosh \gamma + Z_0 \sinh \gamma}{Z_0 \cosh \gamma + Z_R \sinh \gamma} \right)$$

$$Z_{sc} = Z_0 \tanh \gamma$$

$$\underset{\lim Z_R \to \infty}{Z_{oc}} = \frac{Z_0}{\tanh \gamma}$$

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$Z_0 = \sqrt{Z_{oc} Z_{sc}}$$

FILTER FUNDAMENTALS:- PASS AND STOP BANDS:

The propagation constant $\gamma = \alpha + j\beta$ being a function of frequency can supply information on the ability of the filter to perform as desired.

If $\alpha = 0$ or $I_1 = I_2$, then there is no attenuation, only a phase shift, in transmitting a signal through the filter and operation is in a passband of frequencies.

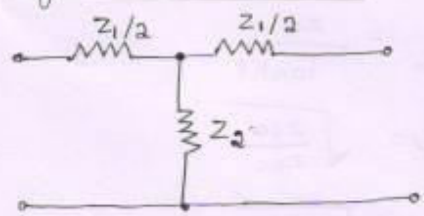
When α has a positive value, then I_2 is smaller in magnitude than I_1, attenuation has occured and operation is in an attenuation or stop band of frequencies.

6.2

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2}+j\frac{\beta}{2}\right)$$

Symmetrical T network.



$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad ; \quad \cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0$$

$\sin\frac{\beta}{2} = 0$; $\beta = n\pi$ where $n = 0, 2, 4, \ldots$

Since $\cos\beta/2 = 1$, then $\sinh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ and the attenuation is $\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$.

Thus the condition that $|Z_1/4Z_2| > 0$ implies a stop or attenuation band of frequencies.

Two conditions are:-

I. $\sinh\frac{\alpha}{2} = 0$;

$$\alpha = 0; \quad \beta \neq 0; \quad \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

II $\cos\frac{\beta}{2} = 0$; $\therefore \sin\frac{\beta}{2} = \pm 1$ and

$$\alpha \neq 0; \quad \beta = (2n-1).\pi; \quad \cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

6.3

The phase angle $\beta = 2\sin^{-1}\sqrt{\frac{z_1}{4z_2}}$; $\alpha = 2\cosh^{-1}\sqrt{\frac{z_1}{4z_2}}$.

CUTOFF FREQUENCIES: The frequencies at which the network changes from a pass network to a stop network are called cutoff frequencies.

$$\frac{z_1}{4z_2} = 0 \text{ or } z_1 = 0.$$

$$\text{where } \frac{z_1}{4z_2} = -1 \text{ or } z_1 = -4z_2.$$

z_1 & z_2 are opposite types of reactance.

BEHAVIOR OF THE CHARACTERISTIC IMPEDANCE:

For a Symmetrical T network

$$z_{0T} = \sqrt{z_1 z_2\left(1+\frac{z_1}{4z_2}\right)}$$

Expression for the characteristic impedance is

$$z_{0T} = \sqrt{-x_1 x_2\left(1+\frac{x_1}{4x_2}\right)}$$

$$z_{0\pi} = \frac{z_1 z_2}{z_{0T}}.$$

The ratio $x_1/4x_2$ will be real and positive and the characteristic impedance will be a pure reactance in this attenuation region.

CONSTANT K FILTERS:

LOW PASS FILTERS:

If z_1 and z_2 of a reactance network are unlike reactance arms then

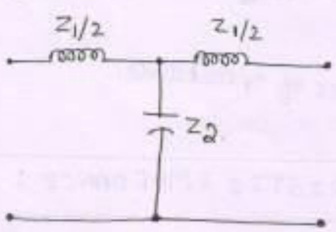
$$z_1 z_2 = k^2$$

where k is a constant independent of frequency.

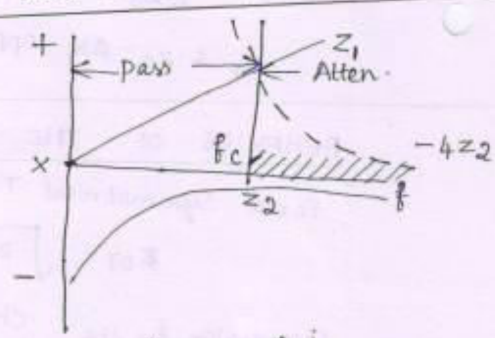
$$Z_1 = j\omega L, \quad Z_2 = -j/\omega C.$$

$$Z_1 Z_2 = \frac{L}{C} = R_K^2.$$

a) Low pass Filter Section.



b) Reactance curves demonstrating that (a) is a Low pass Section or has a Pass band between $Z_1 = 0$ & $Z_1 = -4Z_2$

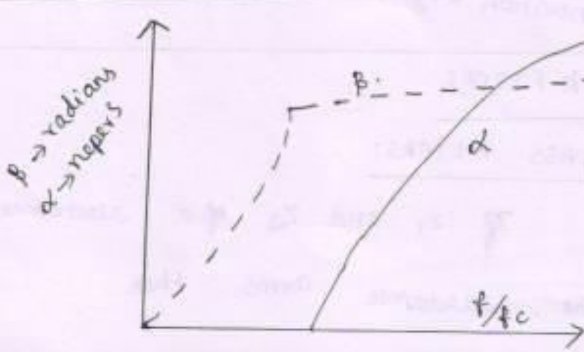


The cutoff Frequency f_c at $Z_1 = -4Z_2$; $j\omega_c L = \frac{4j}{\omega_c C}$.

$$f_c = \frac{1}{\pi\sqrt{LC}}.$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{j\omega\sqrt{LC}}{2} \quad ; \quad \sinh\frac{\gamma}{2} = j\frac{f}{f_c}.$$

variation of α and β with Frequency for the Low pass Section.



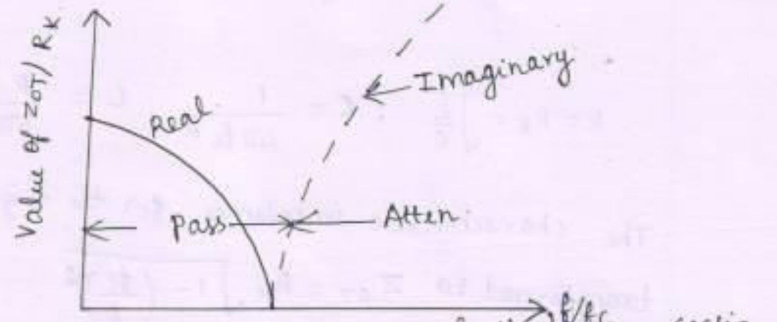
6.5

$$Z_{OT} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} \qquad Z_{OT} = \sqrt{\frac{L}{C}\left(1-\frac{\omega^2 LC}{4}\right)}$$

$$Z_{OT} = \sqrt{\frac{L}{C}\left[1-\left(\frac{f}{f_c}\right)^2\right]} = R_K\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$Z_1 = -4Z_2$; $\omega_c L = \frac{4}{\omega_c C}$; $\pi^2 f_c^2 LC = 1$; $C = \frac{1}{\pi f_c R}$,

$L = \frac{R}{\pi f_c}$.

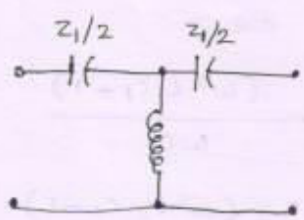


Variation of Z_{OT}/R_K with frequency for the low pass section.

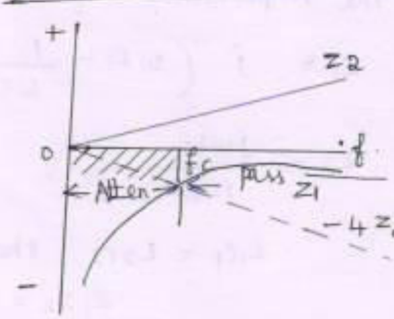
HIGH PASS BAND FILTERS:

If the positions of inductance and capacitance are interchanged to make $Z_1 = -j/\omega C$ & $Z_2 = j\omega L$, then $Z_1 Z_2 = K^2$ and the filter design obtained will be of the constant K-type.

a) High pass filter section



b) Reactance curves demonstrating that (a) is a high pass section or has a pass band between $Z_1 = 0$ & $Z_1 = -4Z_2$



$$Z_1 = -4Z_2 \text{ or } \frac{-j}{\omega_c C} = -j4\omega_c L; \quad 4\omega_c^2 LC = 1$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

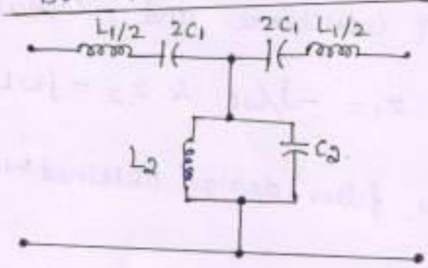
$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{-\frac{1}{4\omega^2 LC}} = \frac{-j}{2\omega\sqrt{LC}} = -j\frac{f_c}{f}$$

$$R = R_K = \sqrt{\frac{L}{C}}; \quad C = \frac{1}{4\pi f_c R}; \quad L = \frac{R}{4\pi f_c}$$

The characteristic impedance for the high pass filter may transformed to $Z_{0T} = R_K\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

BAND PASS FILTERS :

Band Pass Filter network.



It is desirable to pass a band of Frequencies and to attenuate frequencies on both sides of the passband.

condition of equal resonant Frequencies

$$\omega_0^2 L_1C_1 = 1 = \omega_0^2 L_2C_2; \quad L_1C_1 = L_2C_2$$

The impedances of the arms are

$$Z_1 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) = j\frac{(\omega^2 L_1C_1 - 1)}{\omega C_1}$$

$$Z_2 = \frac{j\omega L_2}{1-\omega^2 L_2C_2}; \quad Z_1Z_2 = \frac{-L_2(\omega^2 L_2 C_1 - 1)}{C_1(1-\omega^2 L_2C_2)}$$

$L_1C_1 = L_2C_2$ Then

$$Z_1Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = R_K^2$$

6.7

$$f_{\infty 1} = \sqrt{\frac{(f_2 - f_1)^2}{4(1-m^2)} + f_1 f_2} - \frac{f_2 - f_1}{2\sqrt{1-m^2}}$$

$$f_{\infty 2} = \sqrt{\frac{(f_2 - f_1)^2}{4(1-m^2)} + f_1 f_2} + \frac{f_2 - f_1}{2\sqrt{1-m^2}}$$

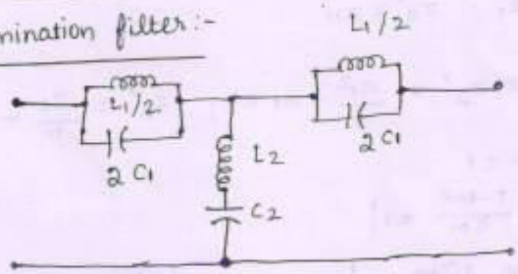
$$f_{\infty 1} f_{\infty 2} = \frac{f_2^2 + 2f_1 f_2 + f_1^2 - 4m^2 f_1 f_2 - f_2^2 + 2f_1 f_2 - f_1^2}{4(1-m^2)}$$

$$= f_1 f_2$$

$$\sqrt{f_{\infty 1} f_{\infty 2}} = f_0$$

BAND ELIMINATION FILTERS:

Band elimination filter:-



$R_k^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$; $f_0 = \sqrt{f_1 f_2}$

At the cut-off frequencies,

$z_1 = -4z_2$; $z_1 z_2 = -4z_2^2 = R_k^2$; $z_2 = \pm j \frac{R_k}{2}$

$$z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = \frac{jR}{2}$$

$L_2 C_2 = \frac{1}{\omega_0^2}$; $1 - \frac{f_1^2}{f_0^2} = \pi f_1 C_2 R$

$$C_2 = \frac{1}{\pi R}\left(\frac{f_2 - f_1}{f_1 f_2}\right)$$

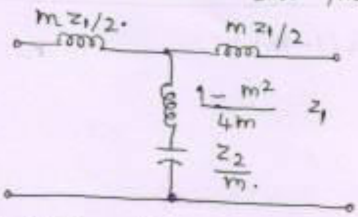
$$f_0 = \sqrt{f_1 f_2} = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

$$L_2 = \frac{R}{4\pi(f_2-f_1)}, \quad L_1 = \frac{R(f_2-f_1)}{\pi f_1 f_2}; \quad C_1 = \frac{1}{4\pi R(f_2-f_1)}$$

m- DERIVED SECTIONS:

m- DERIVED T SECTION:

m-DERIVED LOW PASS FILTER



m- derived section is designed following the design of the prototype T section. The use of a prototype and one or more m- derived sections in series results in a composite filter. m is arbitrary.

$$z_1' = m z_1; \quad z_0' = z_0$$

$$\frac{(m z_1)^2}{4} + m z_1 z_2' = \frac{z_1^2}{4} + z_1 z_2; \quad z_2' = \frac{z_2}{m} + \frac{1-m^2}{4m} z_1$$

$$0 < m < 1$$

$$\left|\frac{z_2}{m}\right| = \left|\frac{1-m^2}{4m} z_1\right|$$

For Low Pass Filter $\frac{1}{2\pi f_\infty m C} = \frac{1-m^2}{4m} 2\pi f_\infty L$

$$f_\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}}; \quad f_c = \frac{1}{\pi\sqrt{LC}}; \quad f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

$$m = \sqrt{1-(f_c/f_\infty)^2} \qquad f_\infty = f_c\sqrt{1-m^2}; \quad m = \sqrt{1-(f_\infty/f_c)^2}$$

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{z_1}{4z_2}\right|} \quad \text{or} \quad \alpha = 2\sinh^{-1}\sqrt{\left|\frac{z_1}{4z_2}\right|}$$

$$z_1 = j\omega L; \quad z_2 = -j/\omega C$$

$$\left|\frac{z_1}{4z_2}\right| = \frac{m\omega L}{4\left[1/m\omega C - \omega L(1-m^2)/4m\right]}$$

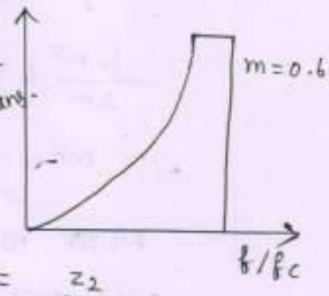
$f_c < f < f_\infty$

$$\alpha = 2\cosh^{-1}\frac{mf/f_c}{\sqrt{1-f^2/f_\infty^2}}$$

For $f_\infty < f$ $\quad \alpha = 2\sinh^{-1}\frac{mf/f_c}{\sqrt{f^2/f_\infty^2 - 1}}$

$$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\sin^{-1}\frac{mf/f_c}{\sqrt{1-(f^2/f_c^2)(1-m^2)}}$$

Variation of phase shift β, for the m-derived filter.



m-DERIVED π SECTION:

The characteristic impedance of π section is

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 (1 + Z_1/4Z_2)}} \; ; \; Z_2' = \frac{Z_2}{m}$$

$$Z_1' = \frac{1}{\frac{1}{mZ_1} + \frac{1}{\frac{4m}{1-m^2} Z_2}} \; ; \; Z_{0T} = R_k\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

Characteristic impedance for a π section is

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 (1 + Z_1/4Z_2)}} \; ; \; Z_{0\pi} = \frac{L/C}{\sqrt{L/C[1-(f/f_c)^2]}}$$

$$Z_{0\pi} = \frac{R_k}{\sqrt{1-(f/f_c)^2}}$$

FILTER CIRCUIT DESIGN:

A composite Low Pass filter is to be terminated in 500 ohms resistance.

$$L = \frac{R}{\pi f_c} = \frac{500}{\pi \times 1000} = 0.159 \text{ henry}.$$

$$C = \frac{1}{\pi f_c R} = \frac{1}{\pi \times 1000 \times 500} = 0.636 \text{ microfarad}$$

The m-derived section to provide high attenuation at $f_\infty = 1065$ cycles may then be designed.

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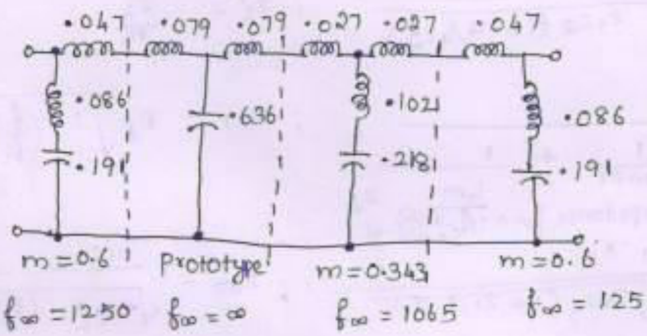
$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{1000}{1065}\right)^2} = \sqrt{1-0.882} = 0.343$$

$$\frac{mL}{2} = \frac{0.343 \times 0.159}{2} = 0.0273\ h.$$

$$\frac{1-m^2}{4m} L = \frac{1-0.118}{1.372} \times 0.159 = 0.102\ h.$$

$$mC = 0.343 \times 0.636 = 0.218 \text{ microfarad.}$$

FILTER AS DESIGNED.



$$m = \sqrt{1-\left(\frac{1000}{1250}\right)^2} = \sqrt{1-0.640} = 0.6$$

Circuit elements. $\frac{mL}{2} = \frac{0.6 \times 0.159}{2} = 0.0474$ henry.

$$\frac{1-m^2}{2m} L = \frac{1-0.36}{1.2} \times 0.159 = 0.086 \text{ henry.}$$

$$\frac{mC}{2} = \frac{0.6 \times 0.636}{2} = 0.191 \text{ micro farad.}$$

FILTER PERFORMANCE :

The filters were designed for 500 ohms resistance termination, and were so used for each measurement. Attenuation measurements were made

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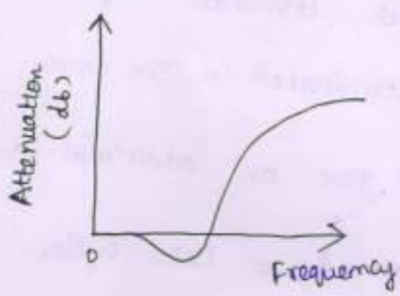
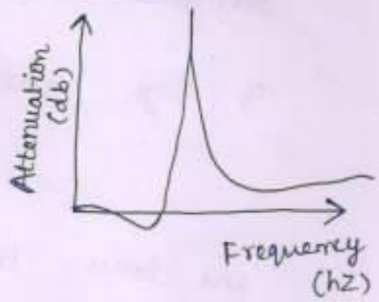
over the pass and stop bands on each of the filter sections. The attenuation of the prototype section, terminated in 500 ohms.

Calculation of attenuation based on pure reactances and the theoretical equation gives 16.7 db attenuation at 1500 cycles or $f/f_c = 1.5$.

a) Attenuation of the prototype: $f_c = 1000$ cycles, $R_K = 500$ ohms.



b) Attenuation of the m-derived section: $m = 0.346$; $f_c = 1000$, $f_\infty = 1065$, $R_K = 500$ ohms.



c) Action of Composite filter

The usually undesirable low α values above f_∞ are also found, the slight rise at 3000 cycles probably being due to increased losses in the coils used.

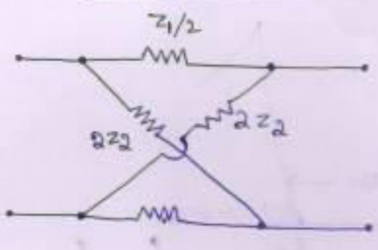
The overall attenuation of the complete filter when the prototype and m-derived sections are combined with terminals half sections having m = 0.6 and terminated in 500 ohms. The terminal half sections give an additional value of high attenuation at f_∞ = 1250 cycles.

Due to probable slight mismatches and losses, there is a small irregularity in the passband, but reasonably sharp cutoff characteristics are obtained.

CRYSTAL FILTERS:

The lattice structure can be shown to have filter properties. Considering the network.

Lattice Filter Section



$$Z_{oc} = \frac{(Z_1/2 + 2Z_2)^2}{2(Z_1/2 + 2Z_2)} = \frac{Z_1}{4} + Z_2$$

$$Z_{sc} = \frac{Z_1 Z_2}{Z_1/2 + 2Z_2} + \frac{Z_1 Z_2}{Z_1/2 + 2Z_2}$$

$$Z_{sc} = \frac{Z_1 Z_2}{(Z_1/4) + Z_2}$$

The characteristic impedance of the lattice section then is

$$Z_{OL} = \sqrt{Z_{oc} Z_{sc}} = \sqrt{Z_1 Z_2}$$

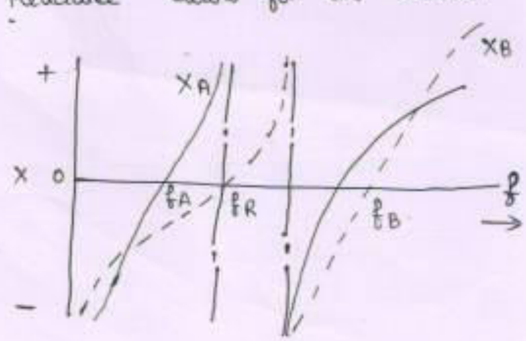
$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{\frac{Z_1}{Z_2}} \left[\frac{1}{1 + Z_1/4Z_2} \right]$$

If f_1 and f_2 are the frequencies of resonance of one of the circuits and f_R is that of the antiresonance, then

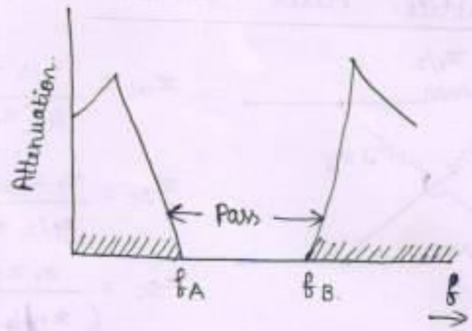
$$f_{1,2} = f_R \sqrt{1 \mp \frac{C_s}{C_p}}$$

The separation of f_1 and f_2 represents two thirds of the pass band and depend on the $\sqrt{C_s/C_p}$ ratio.

a) Reactance curves for the circuit.



Attenuation curves for that circuit.



Thus the use of coils permits the bands to be widened to pass speech frequencies, and crystal filters are quite generally used to separate the various channels in carrier telephone circuits in the range above 50 Kilocycles.